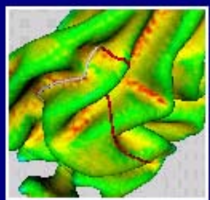
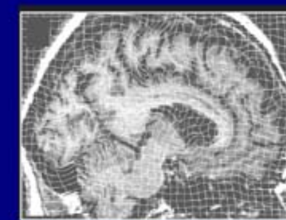
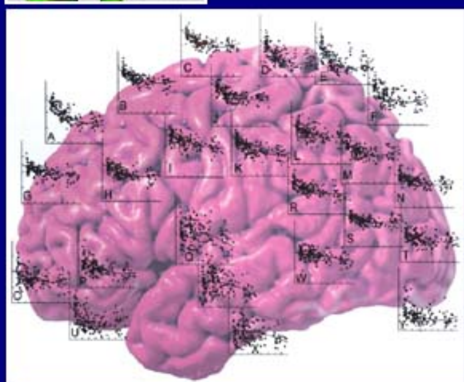
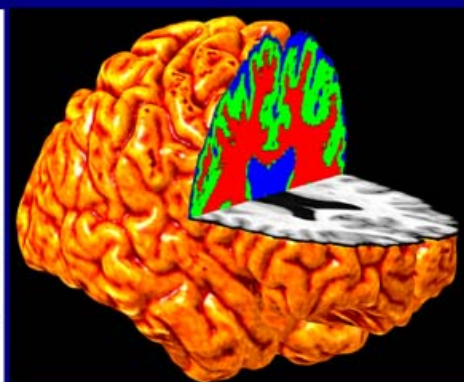
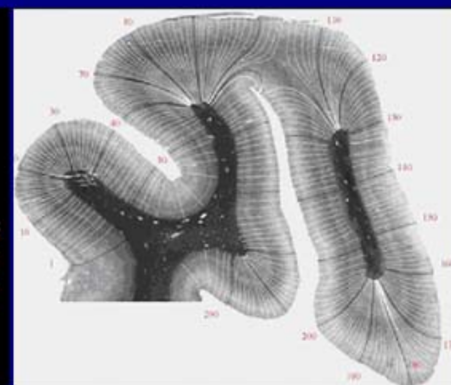
Mathematics, Medical Imaging & Brain Mapping

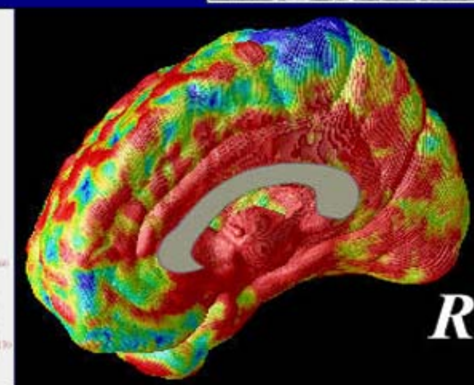




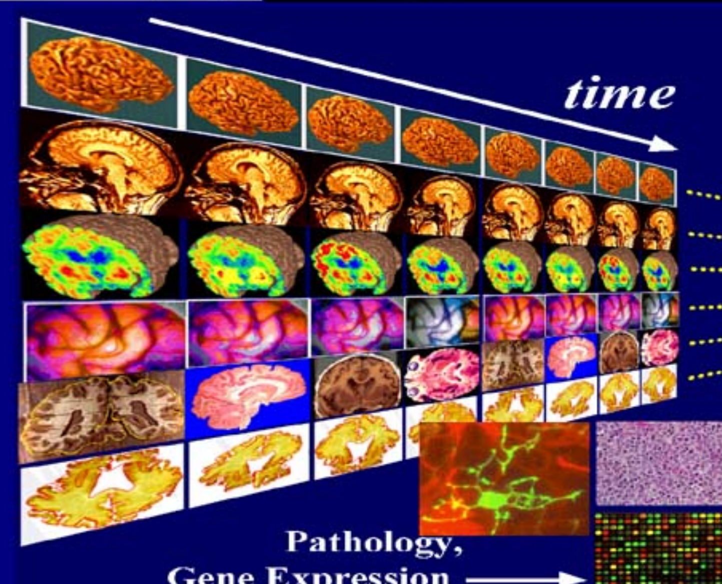






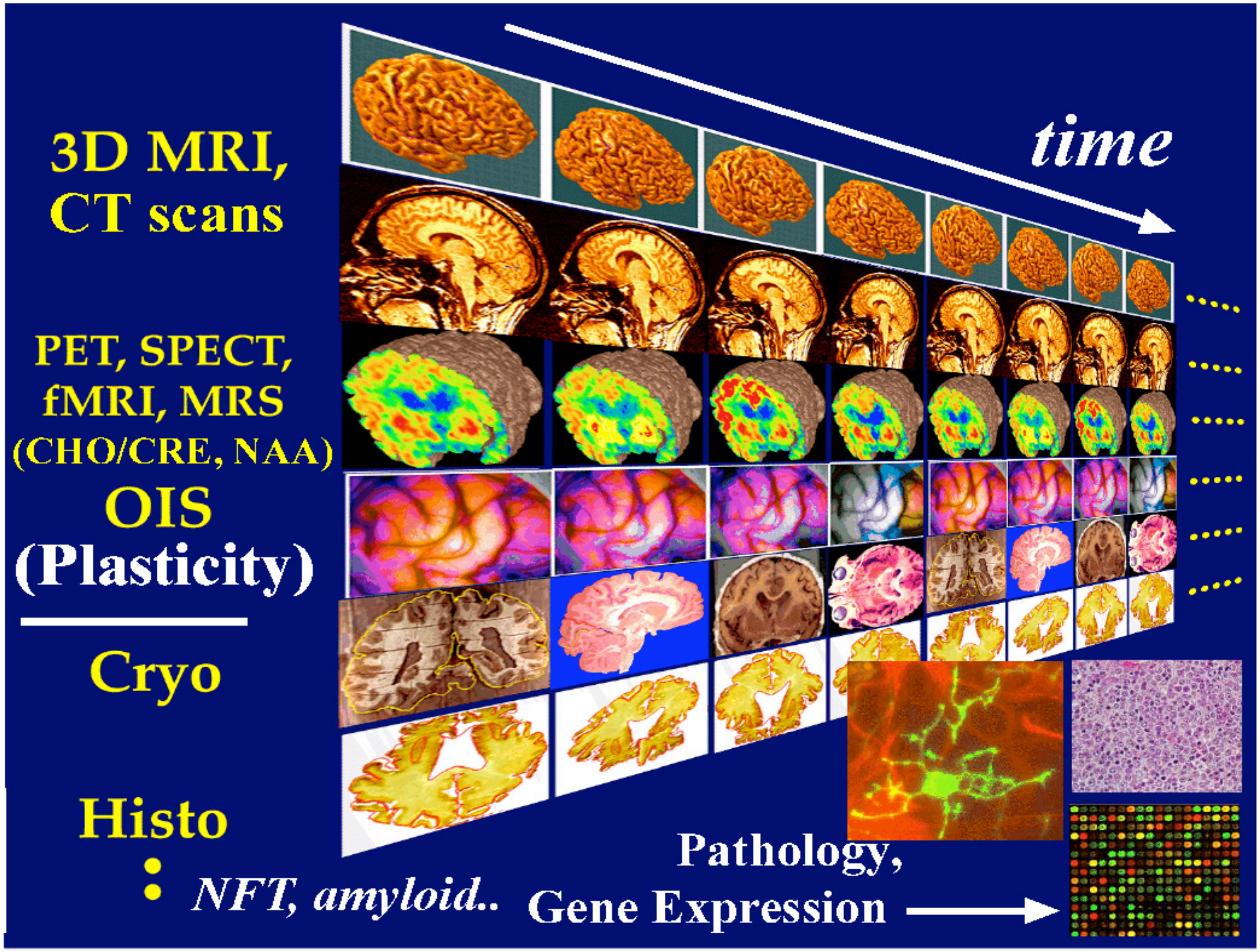


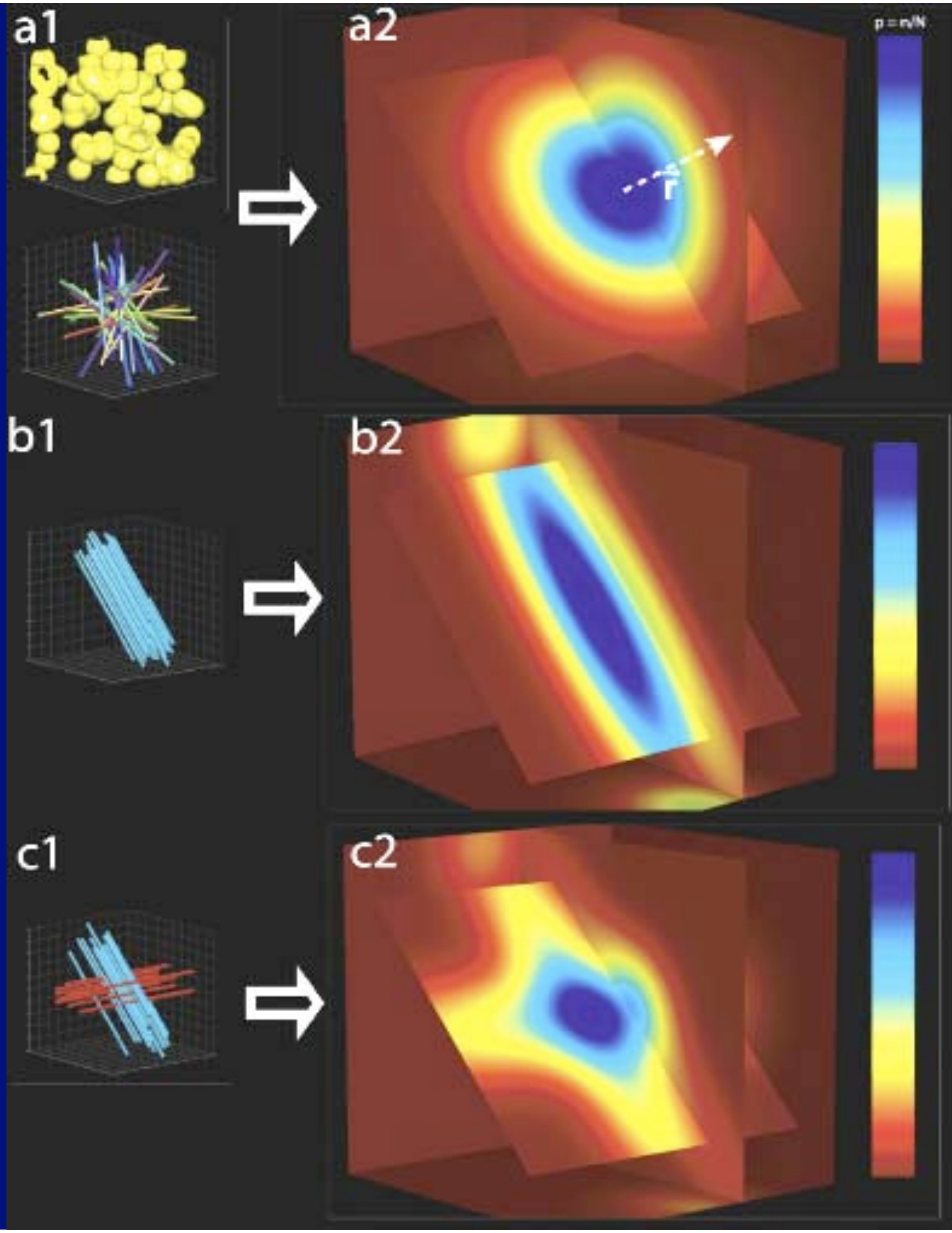
Paul Thompson
Prof. of Neurology
Laboratory of Neuro Imaging
UCLA School of Medicine
Los Angeles, CA
and many colleagues!



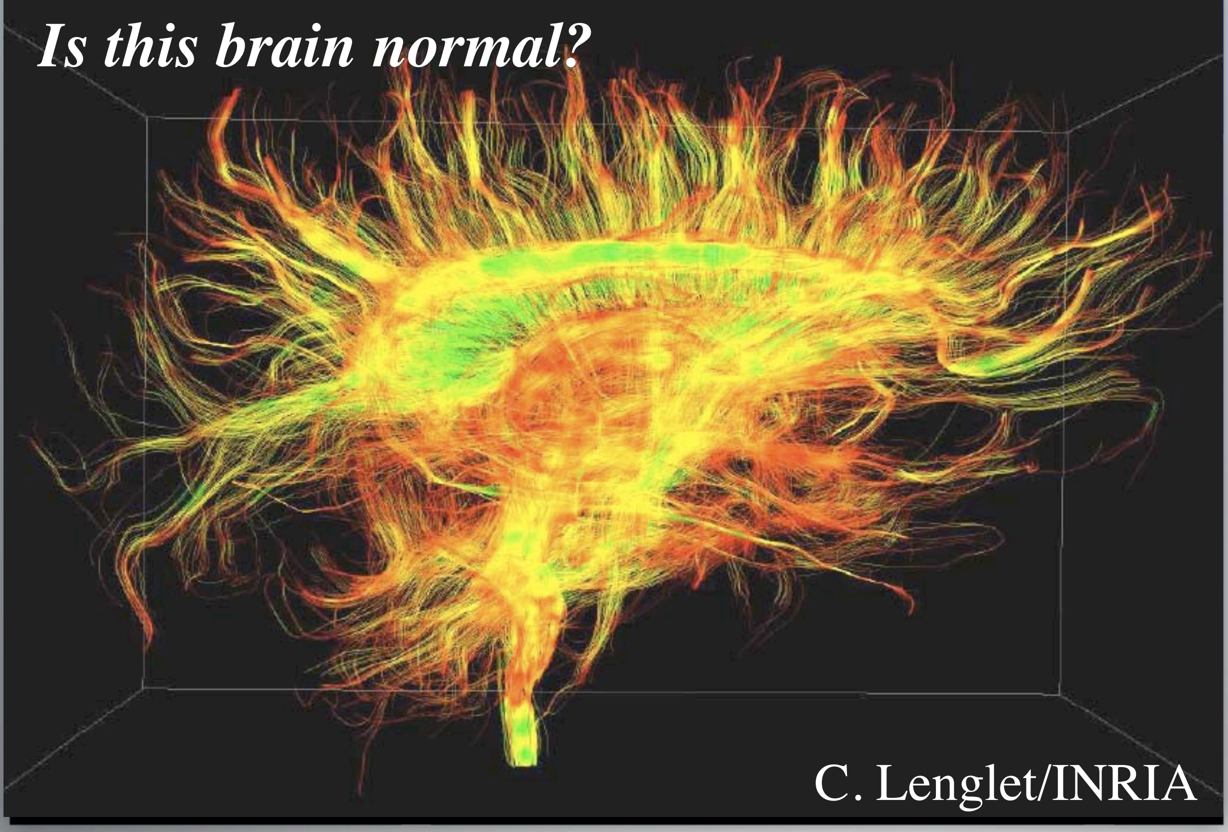
Population-Based Brain Mapping

How does the brain change throughout life? How do we map effects of disease, treatments, genes, on brain structure & function?

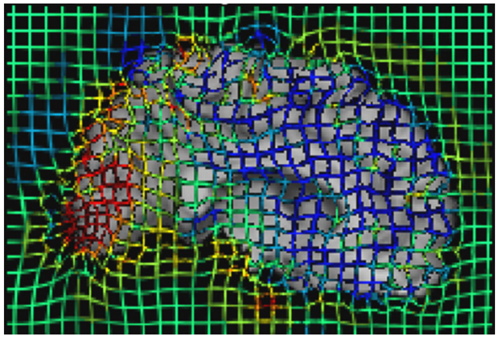



a1
a2
r
b1
b2
c1
c2

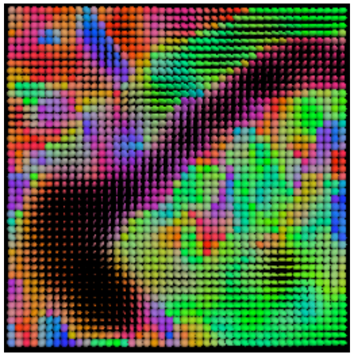
Diffusion Tensor Imaging (DTI) - High-dimensional data at each point; signals lie in Riemannian manifolds; need metrics



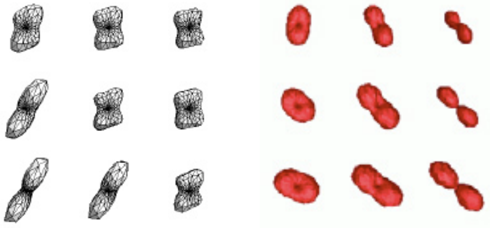
MRI

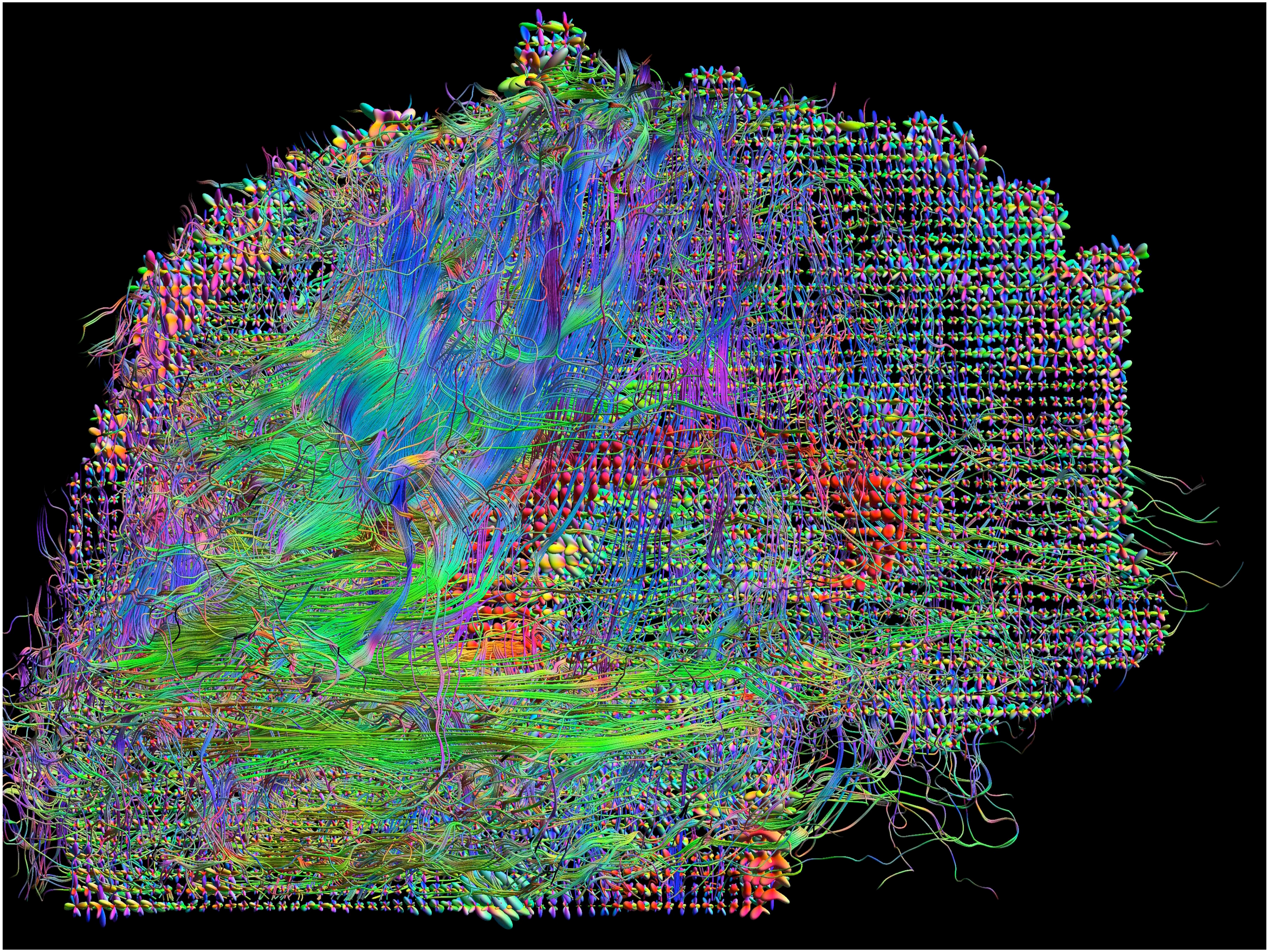


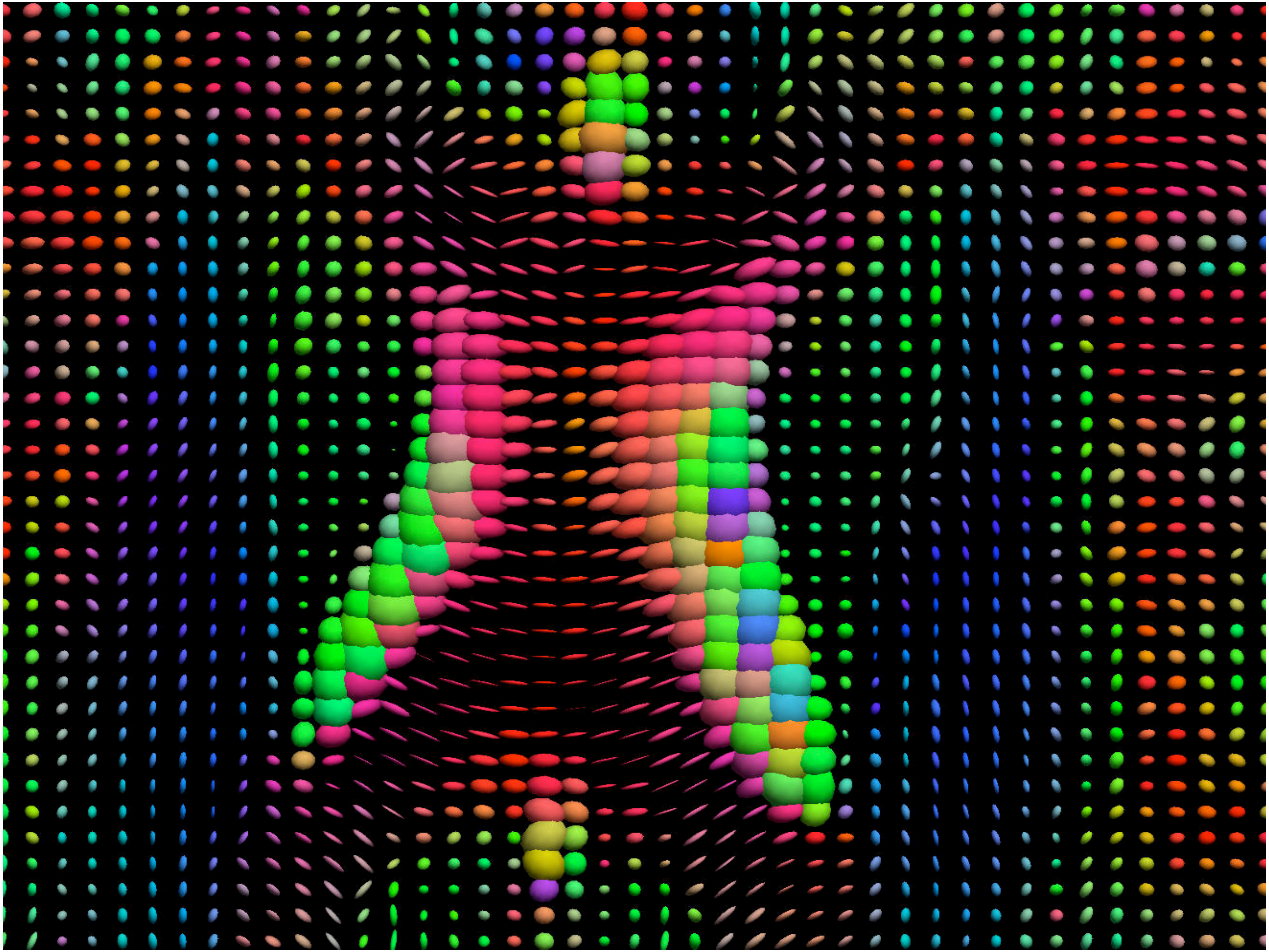
DTI

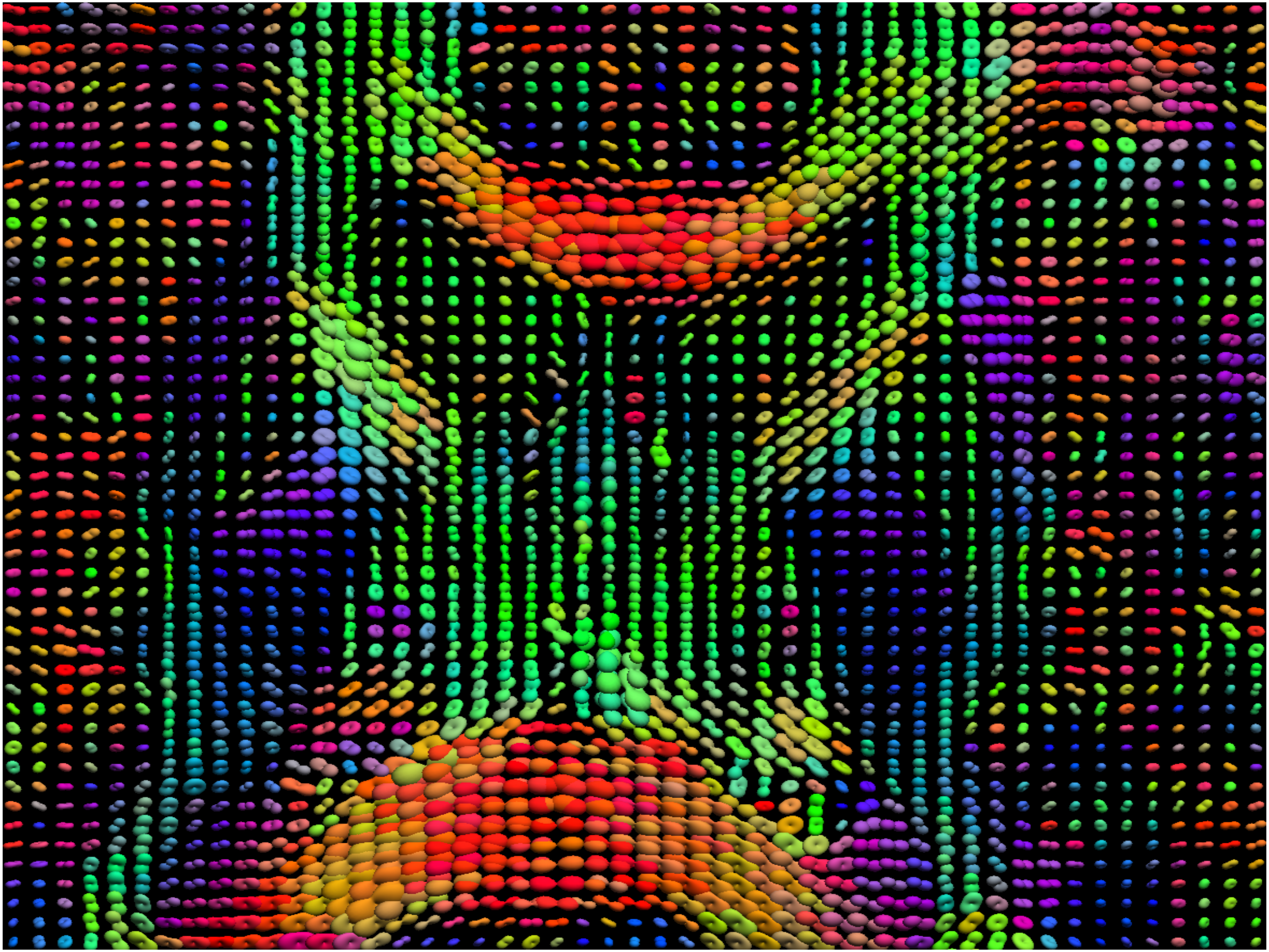


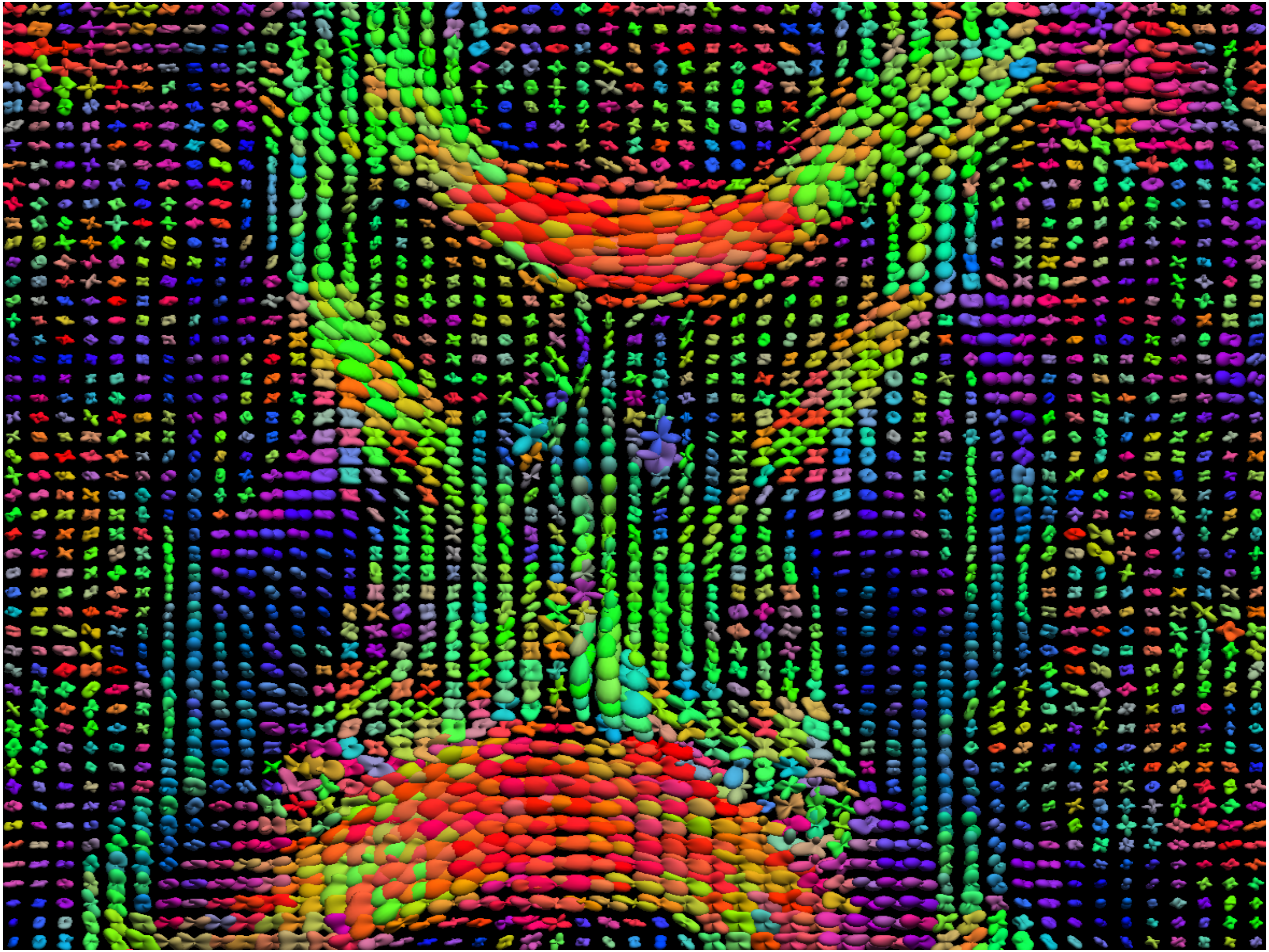
HARDI

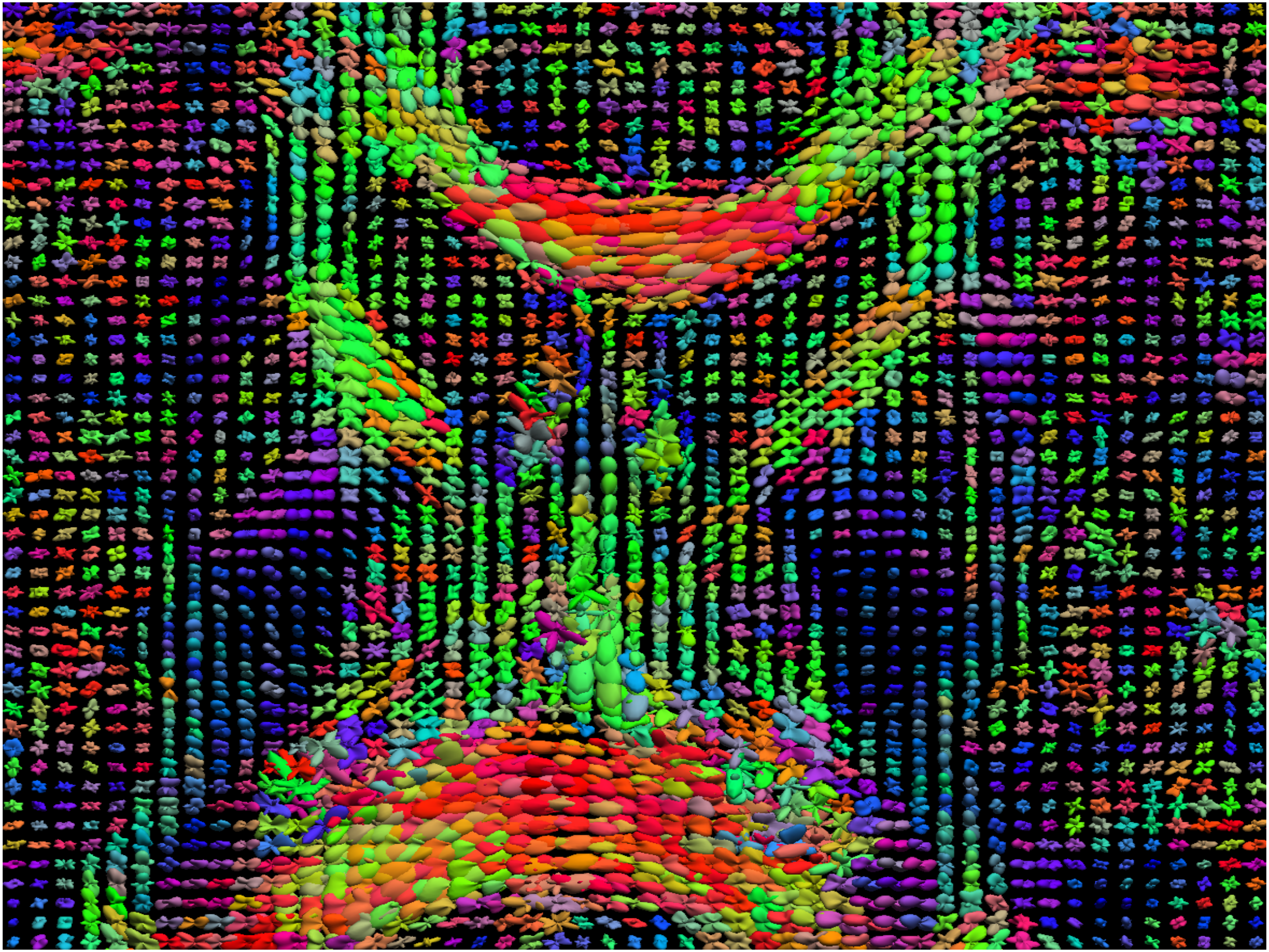


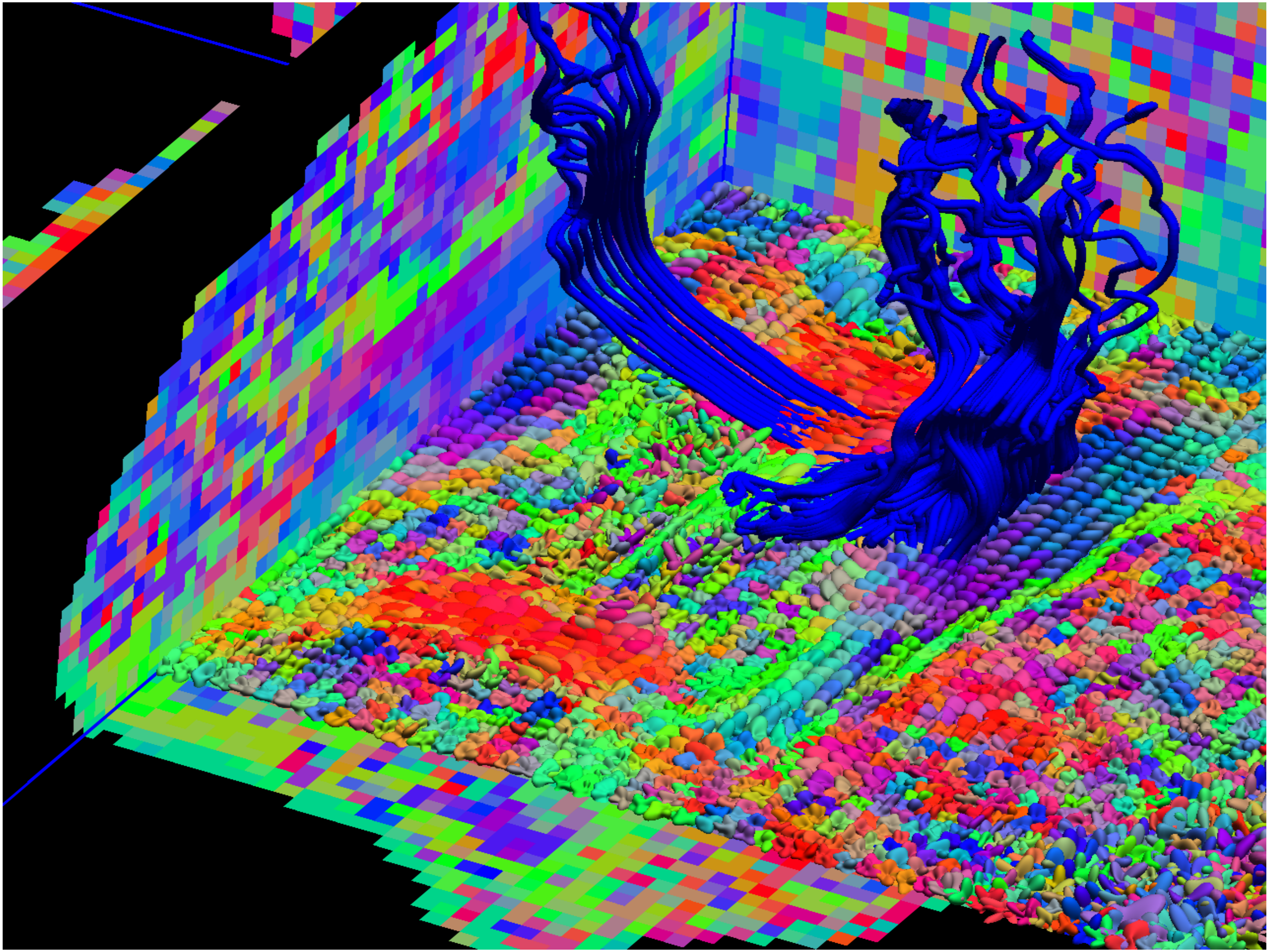


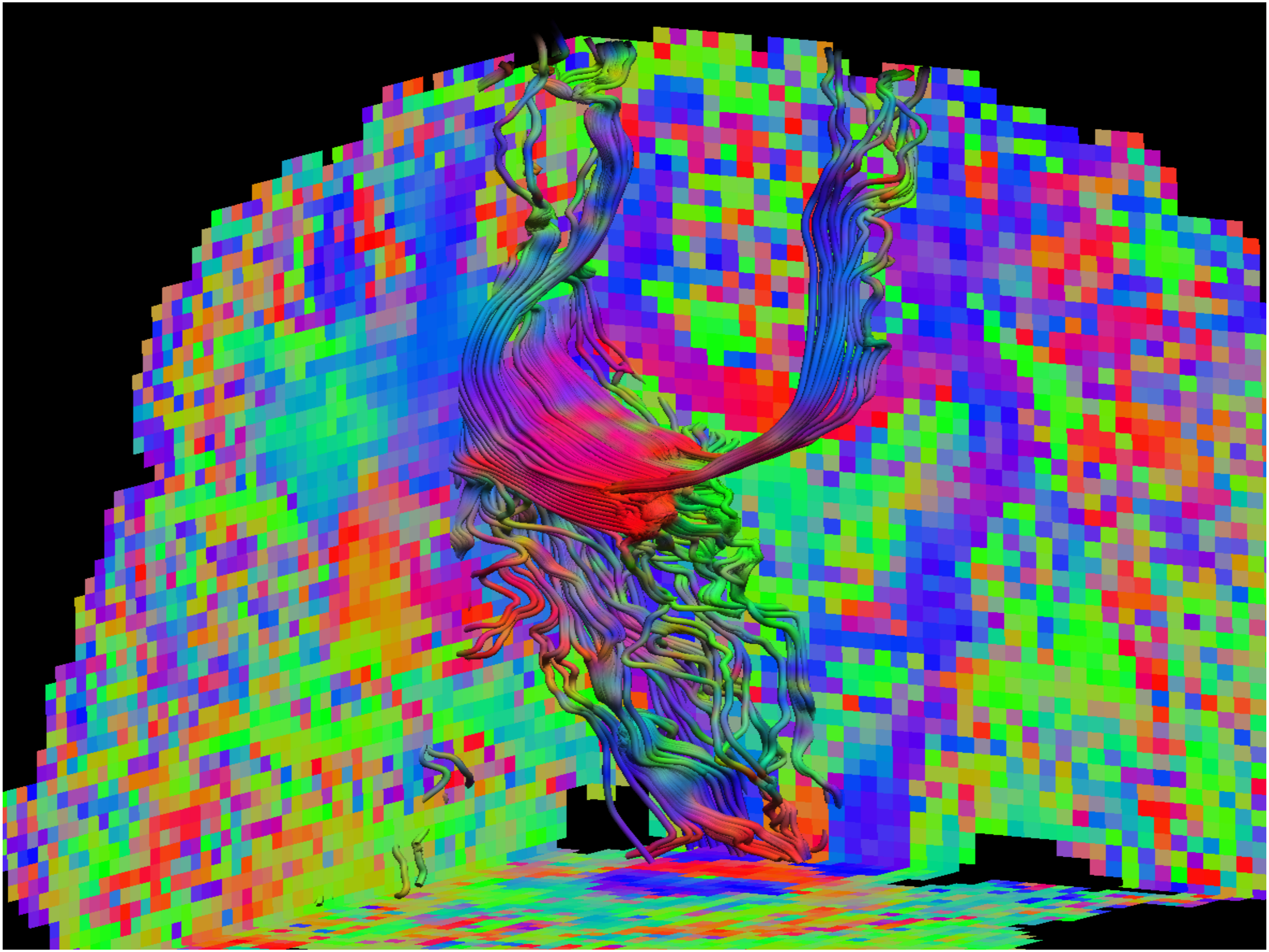


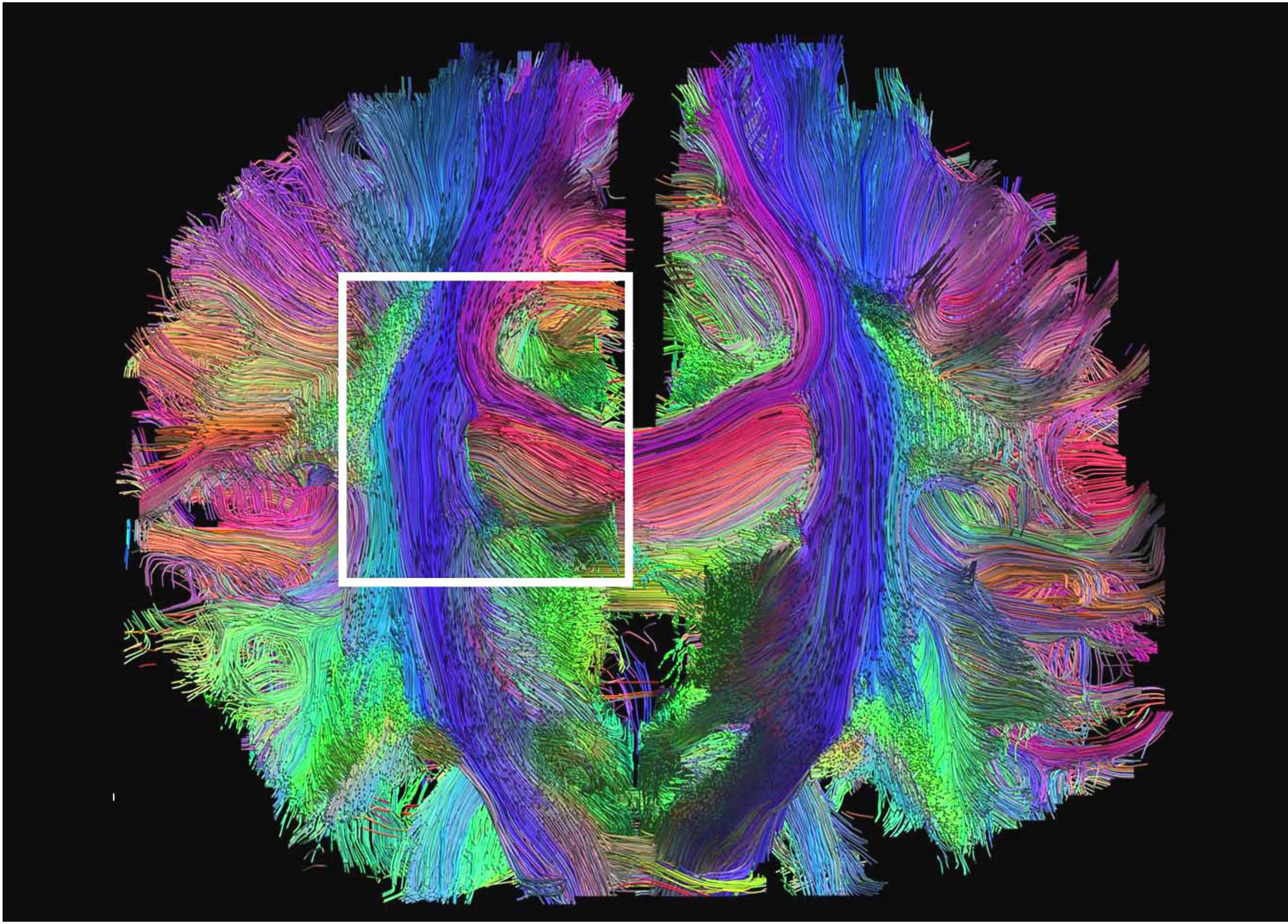


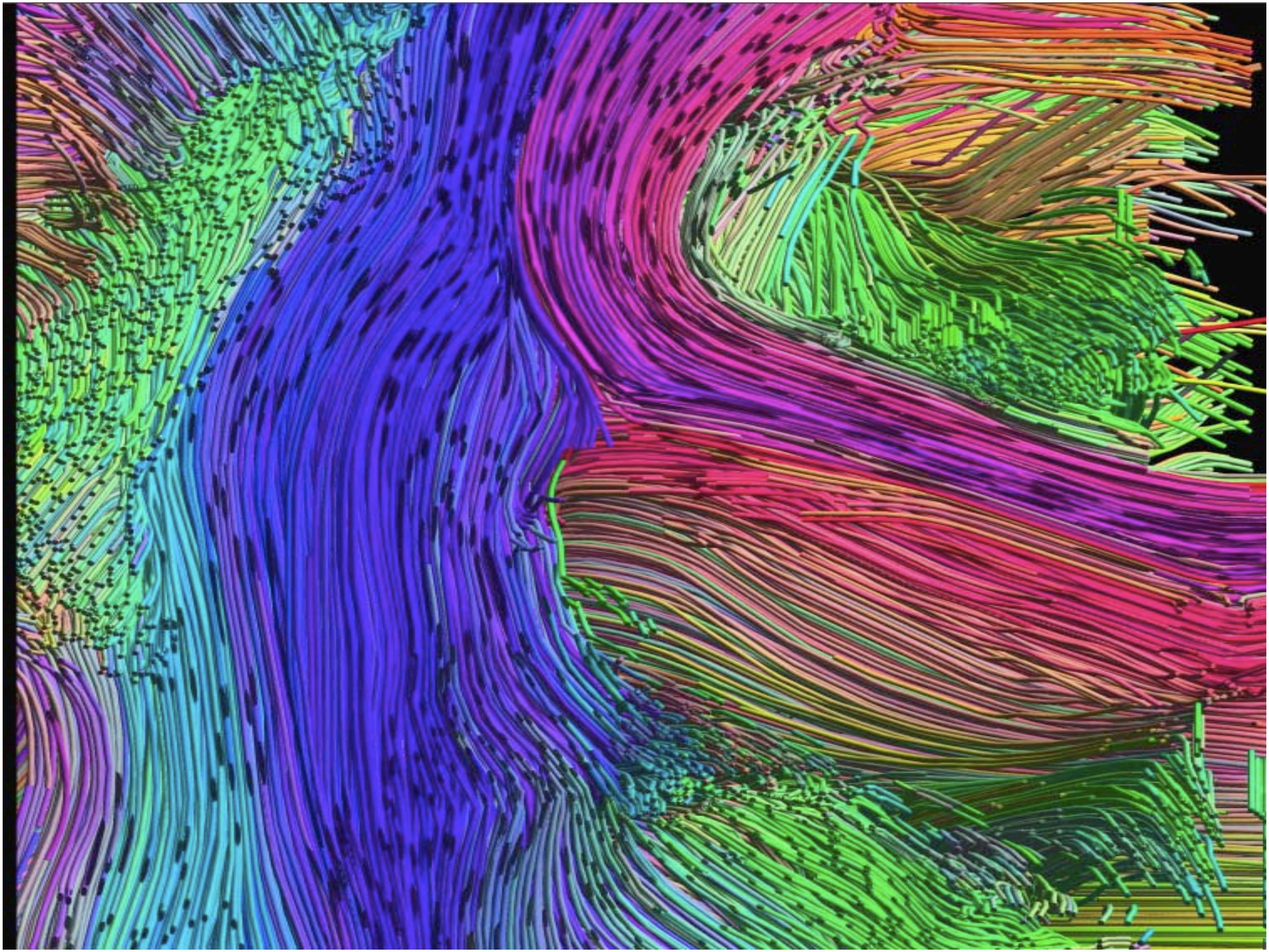


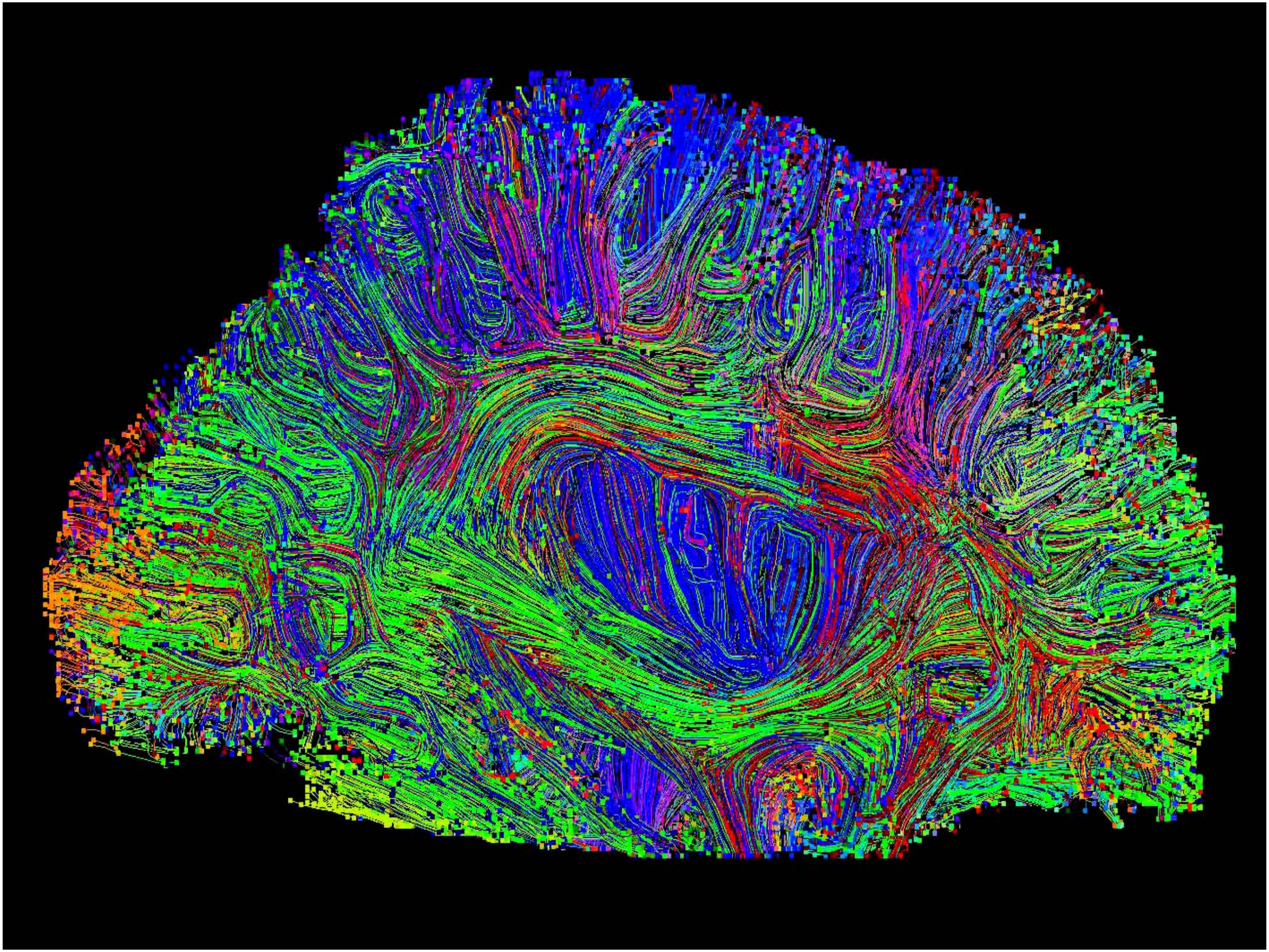


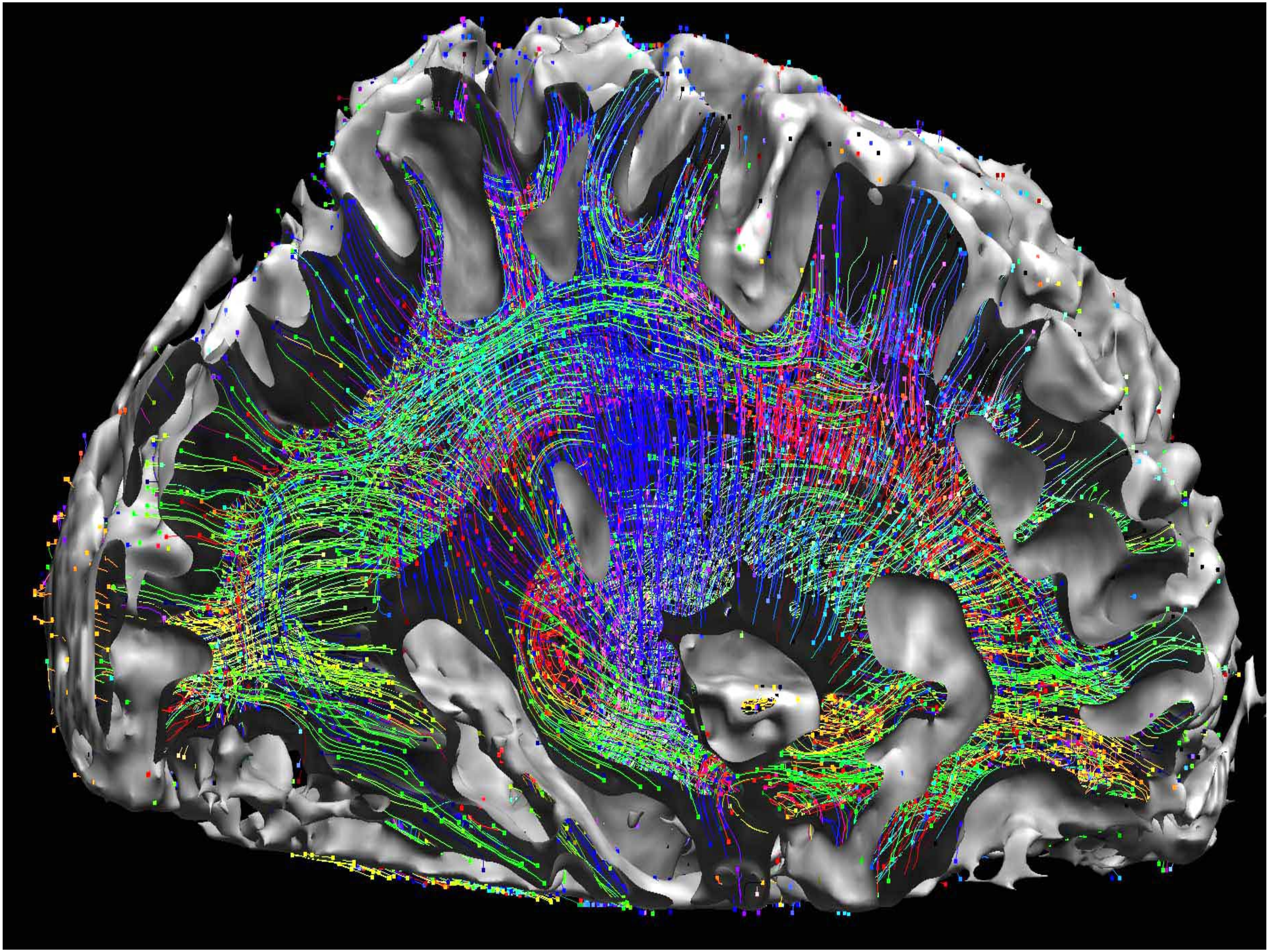


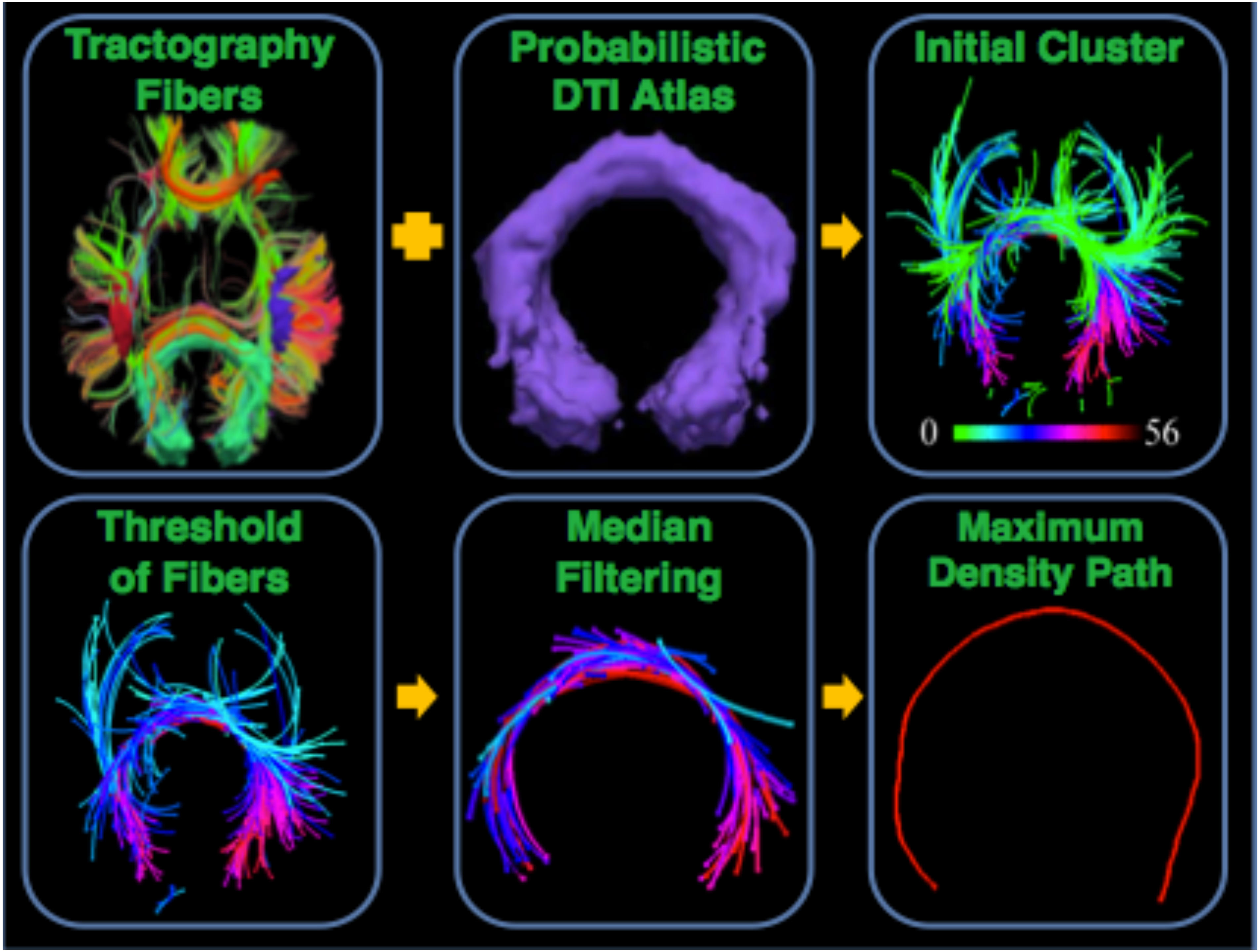


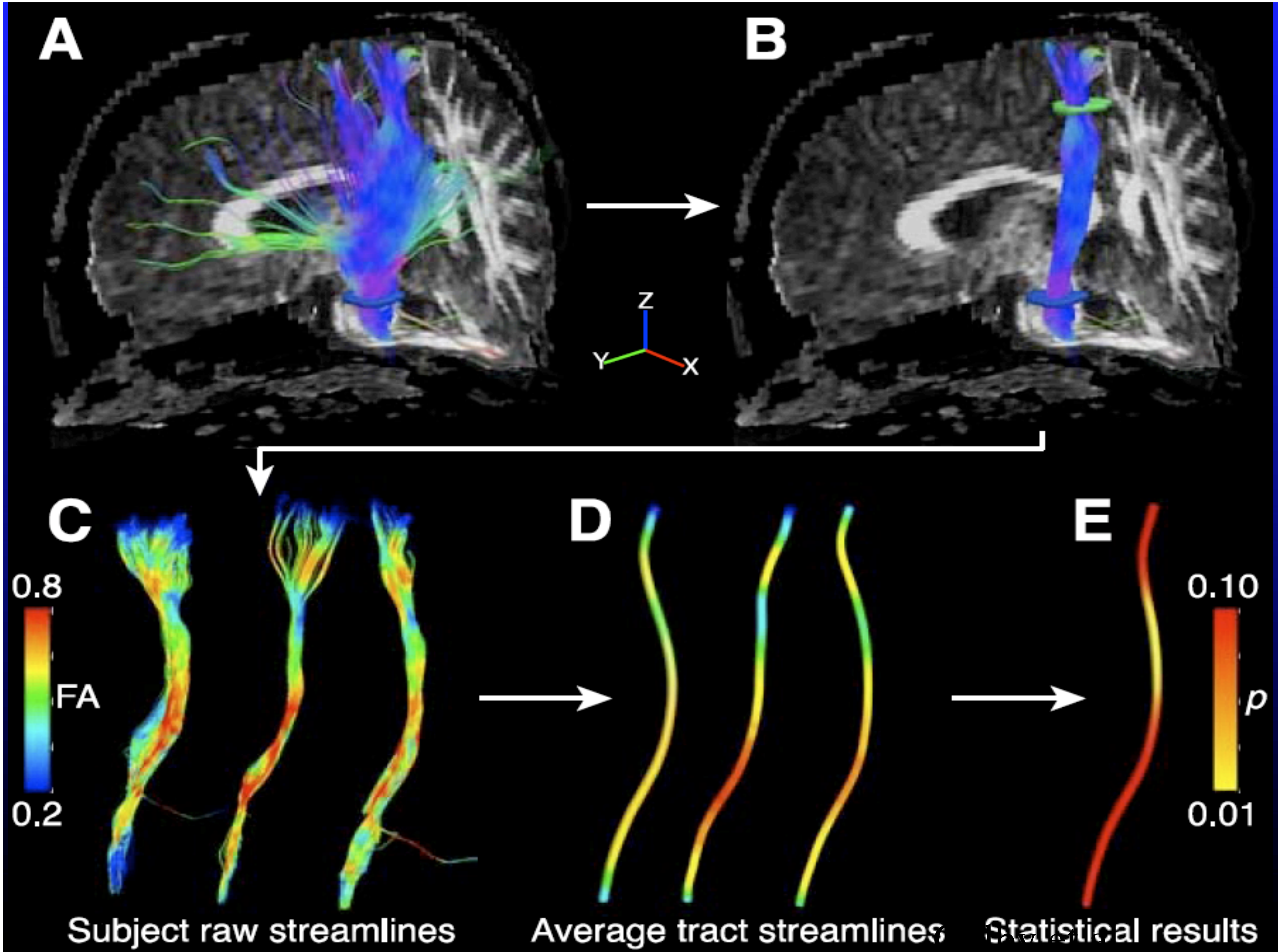






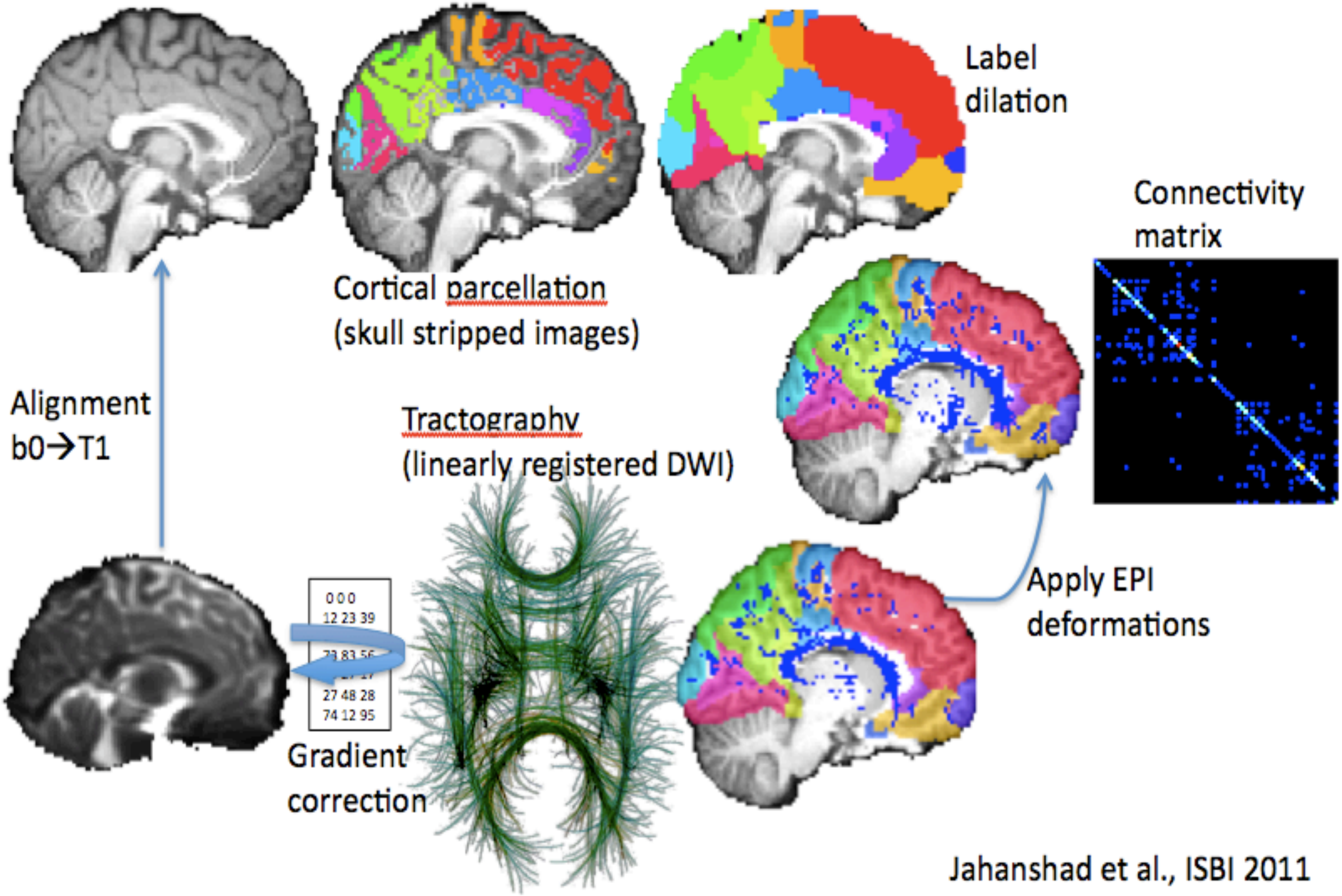



Tractography Fibers
Probabilistic DTI Atlas
Initial Cluster
0
56
Threshold of Fibers
Median Filtering
Maximum Density Path


A
B
Z
Y
X
C
0.8
FA
0.2
D
E
0.10
p
0.01
Subject raw streamlines
Average tract streamlines
Statistical results

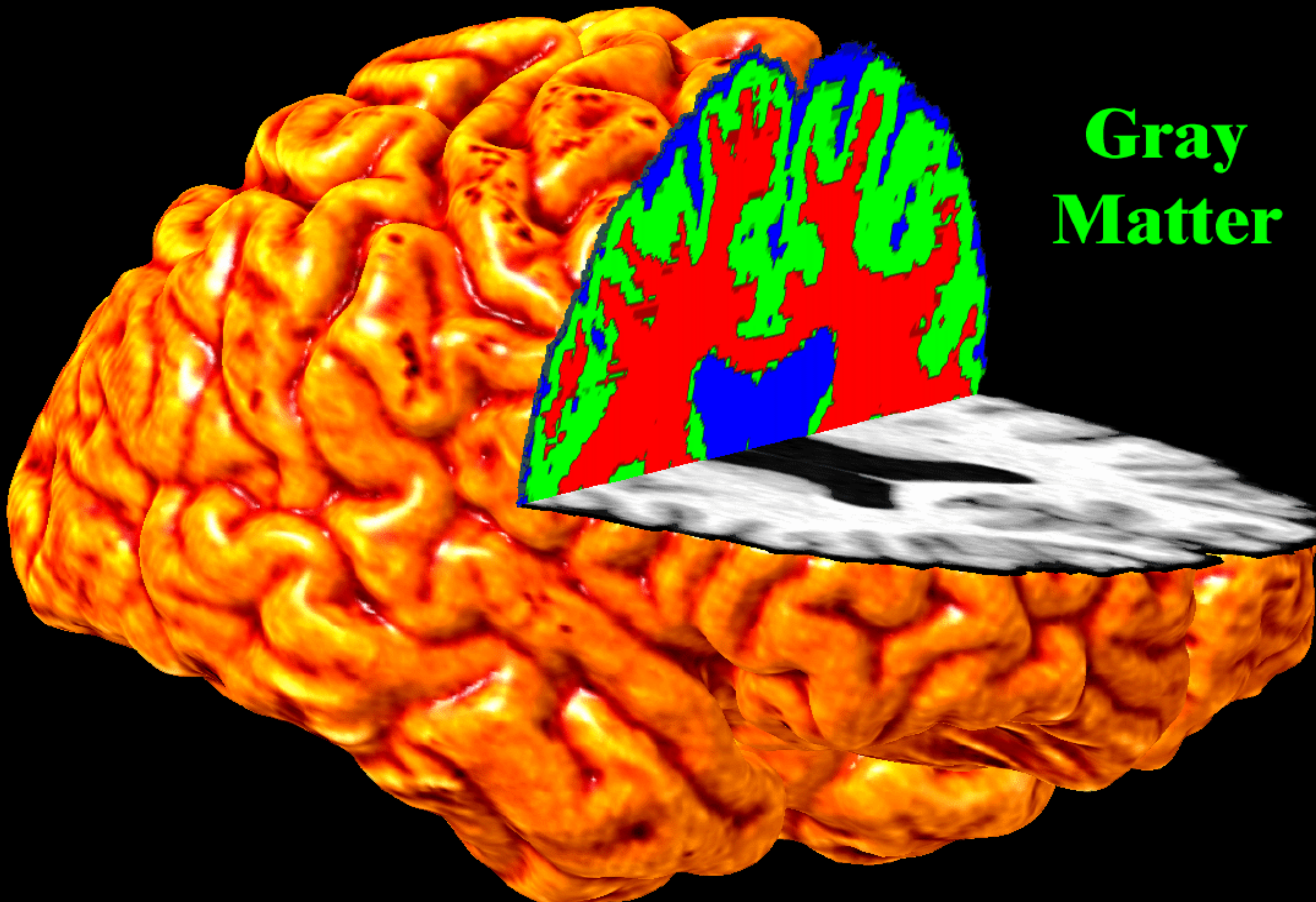
OHBM 2011

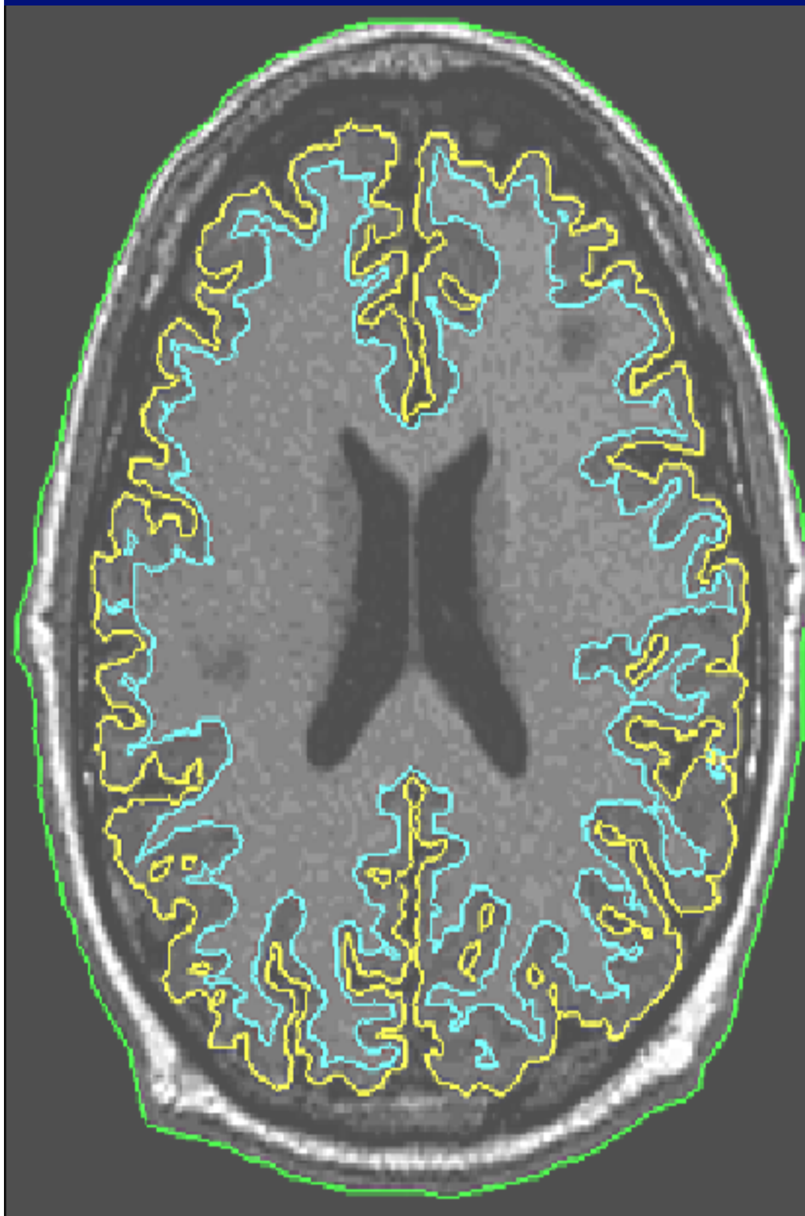
Network Connectivity

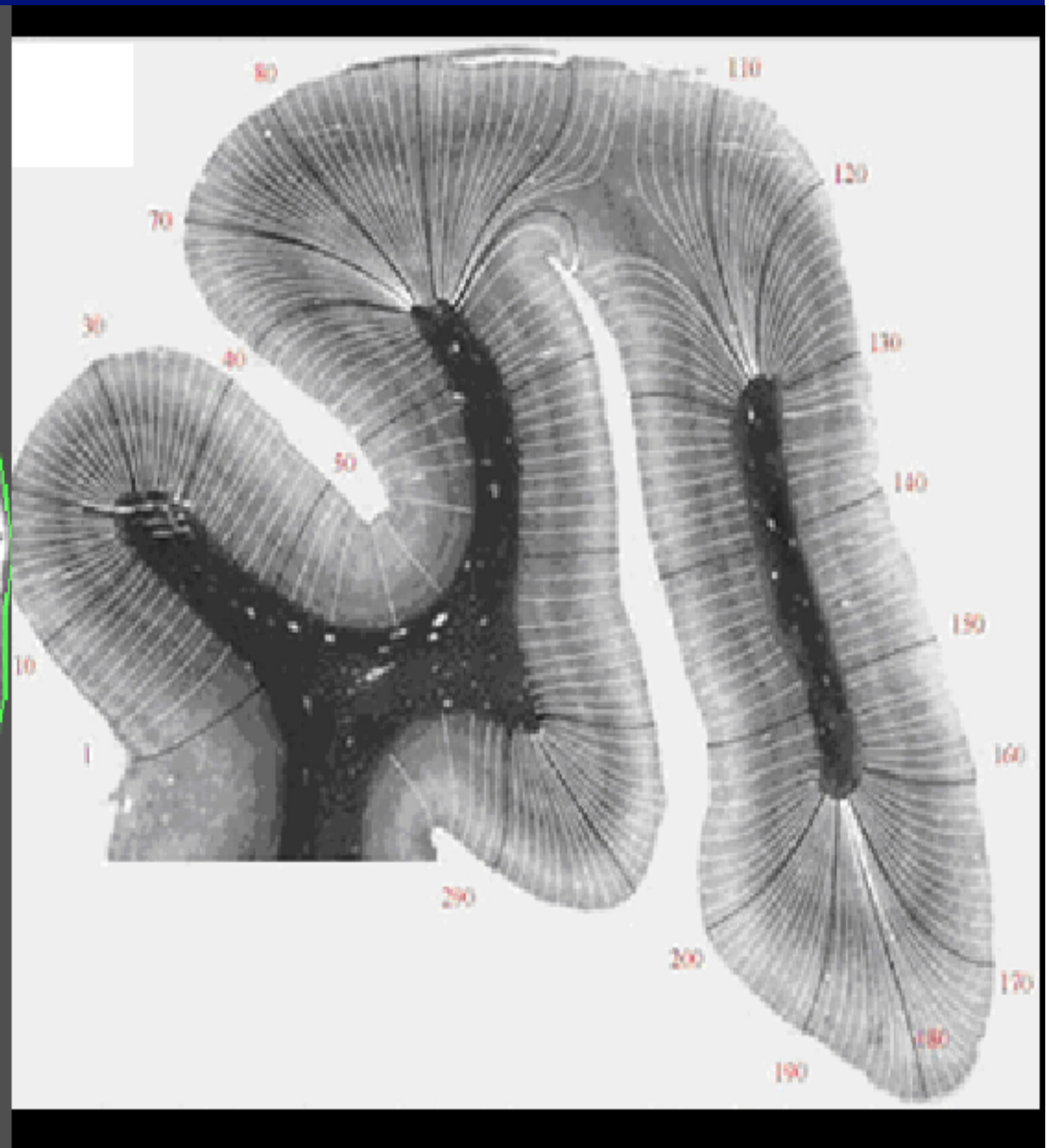


Jahanshad et al., ISBI 2011

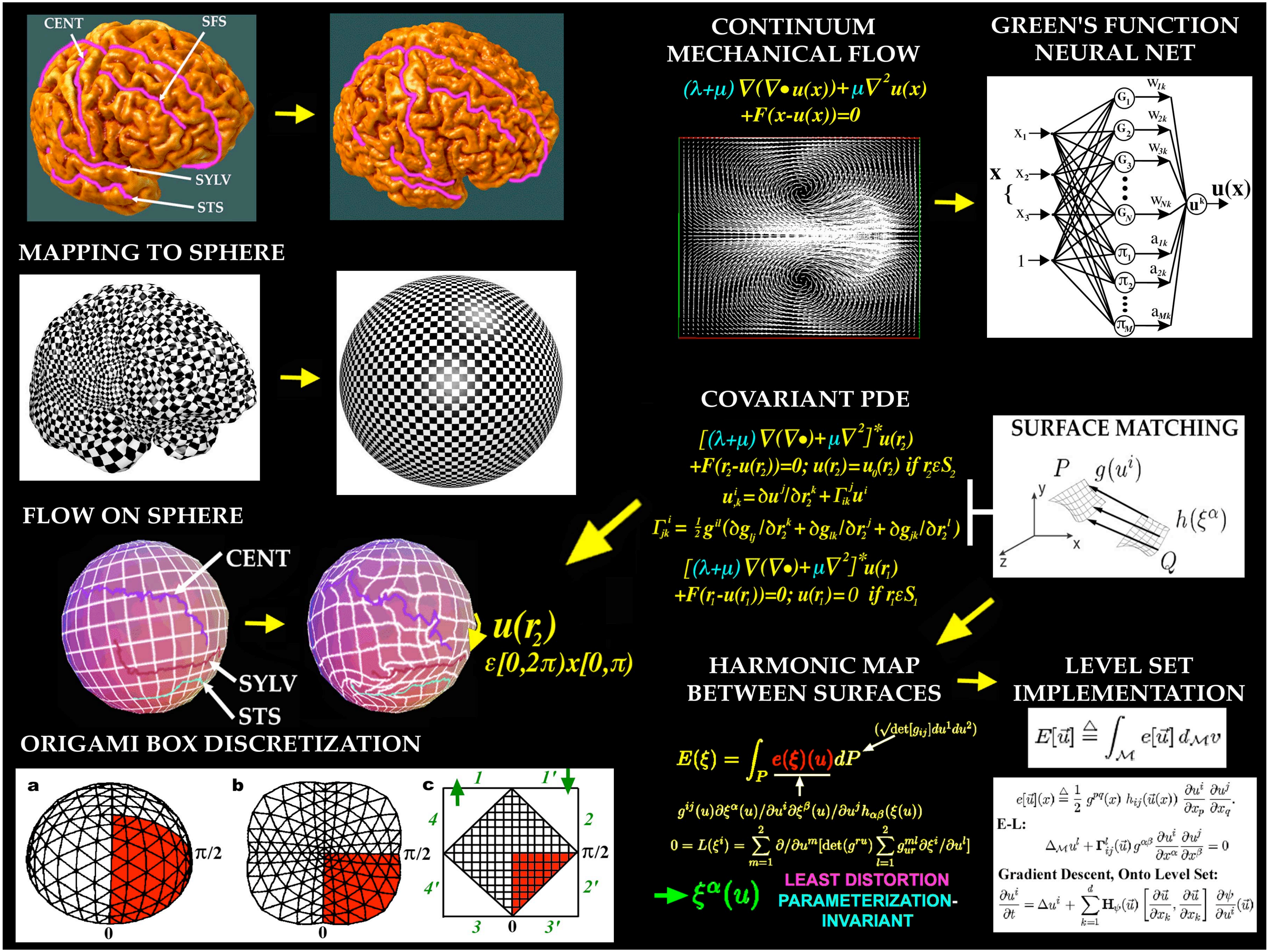
What Happens to Our Gray Matter?





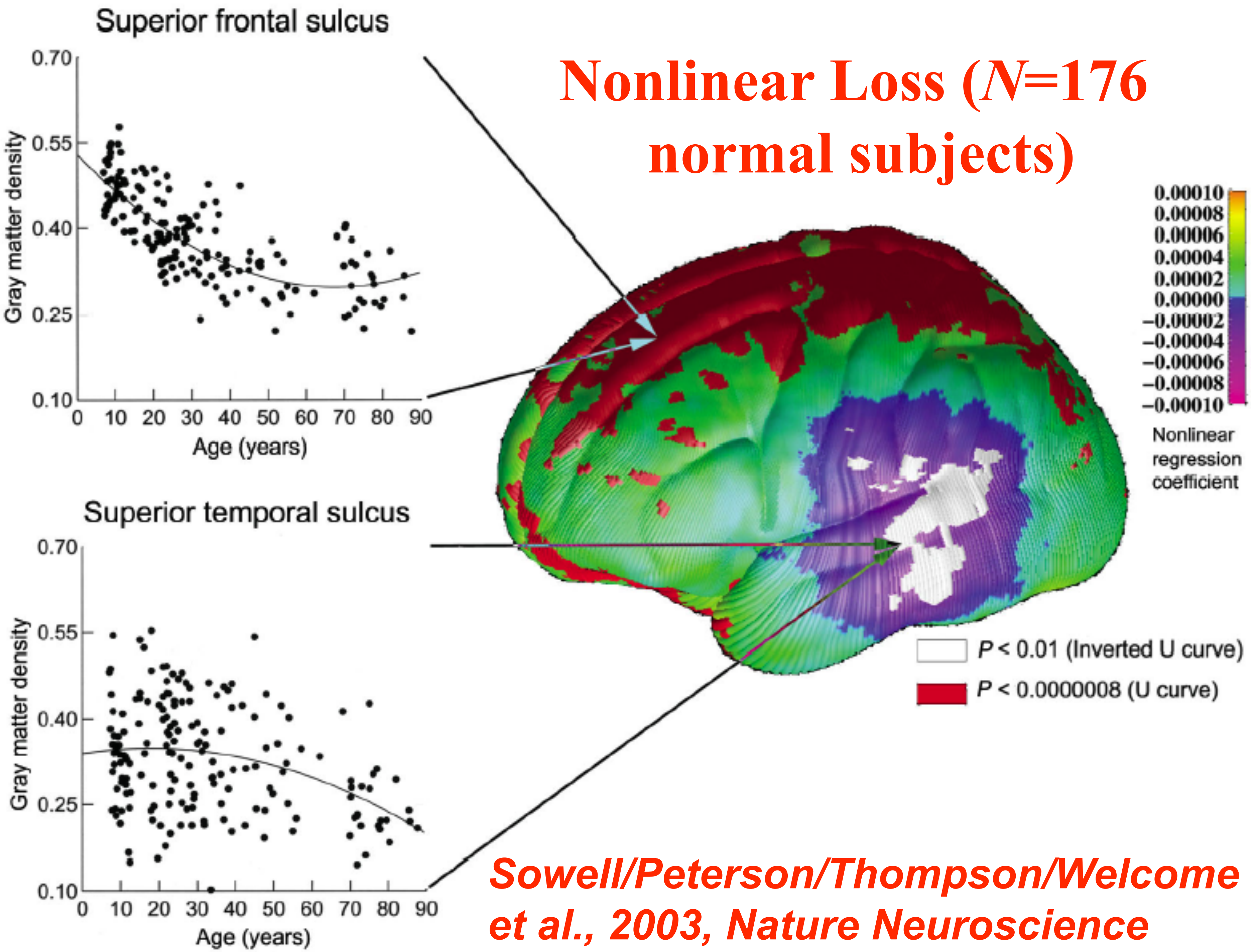


80
110
70
120
30
40
130
50
140
150
10
160
1
290
200
170
180
190

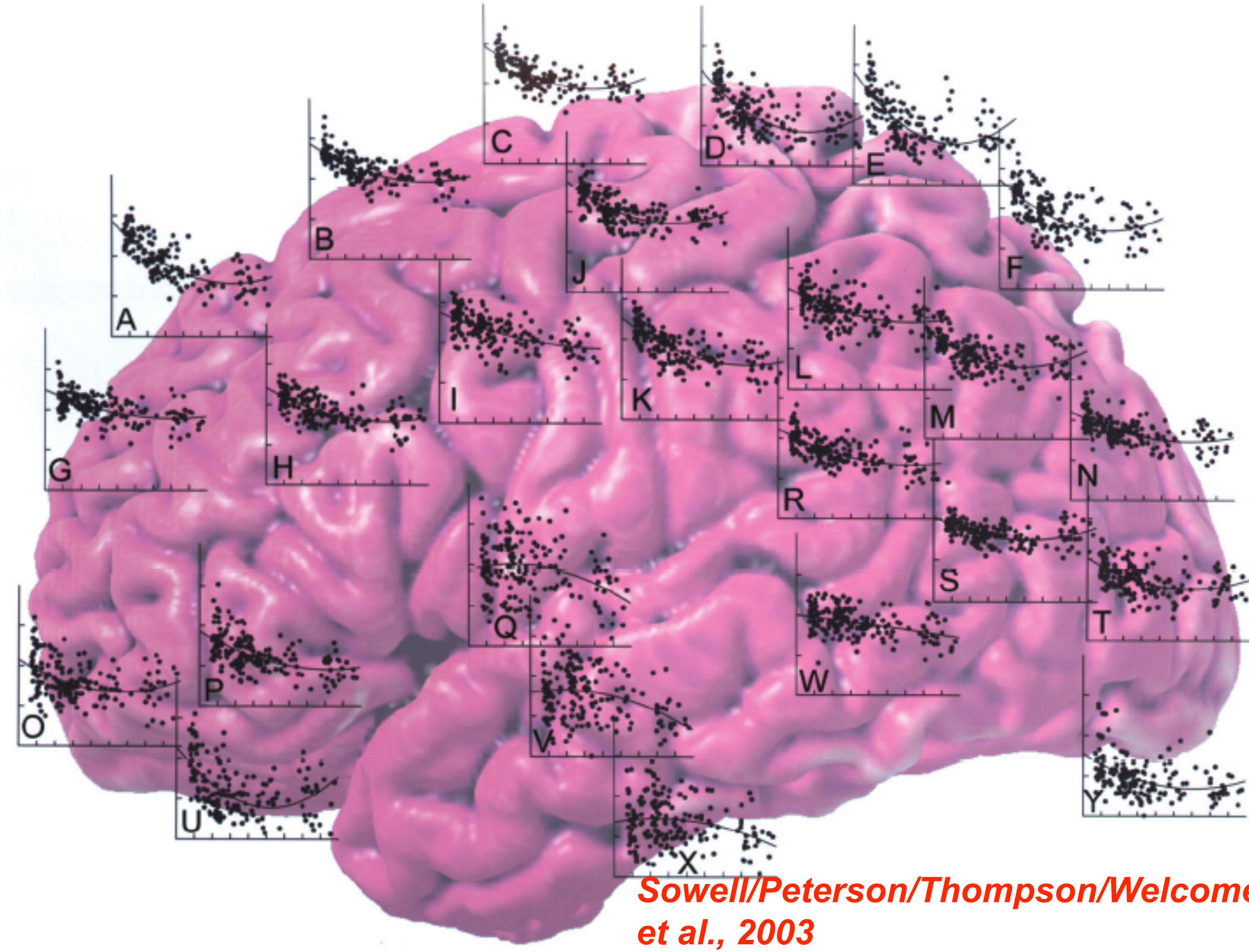


CENT
SFS
SYLV
STS
MAPPING TO SPHERE
FLOW ON SPHERE
CENT
SYLV
STS
$u(r_2)$
$\varepsilon[0,2\pi)x[0,\pi)$
ORIGAMI BOX DISCRETIZATION
a
b
c
$\pi/2$
0
1
1′
2
2′
3
3′
4
4′
CONTINUUM MECHANICAL FLOW
$(\lambda+\mu)\nabla(\nabla\bullet u(x))+\mu\nabla^2 u(x)$
$+F(x-u(x))=0$
GREEN'S FUNCTION NEURAL NET
x_1, x_2, x_3, 1
G_1, G_2, G_3, G_N
π_1, π_2, π_M
W_{1k}, W_{2k}, W_{3k}, W_{Nk}
a_{1k}, a_{2k}, a_{Mk}
u^k
$\mathbf{u(x)}$
COVARIANT PDE
$[(\lambda+\mu)\nabla(\nabla\bullet)+\mu\nabla^2]^* u(r_2)$
$+F(r_2-u(r_2))=0;\ u(r_2)=u_0(r_2)\ if\ r_2\varepsilon S_2$
$u^i_{,k}=\partial u^j/\partial r_2^k+\Gamma^j_{ik}u^i$
$\Gamma^i_{jk}=\frac{1}{2}g^{il}(\partial g_{lj}/\partial r_2^k+\partial g_{lk}/\partial r_2^j+\partial g_{jk}/\partial r_2^l)$
$[(\lambda+\mu)\nabla(\nabla\bullet)+\mu\nabla^2]^* u(r_1)$
$+F(r_1-u(r_1))=0;\ u(r_1)=0\ \ if\ r_1\varepsilon S_1$
SURFACE MATCHING
P $g(u^i)$
$h(\xi^\alpha)$
Q
x y z
HARMONIC MAP BETWEEN SURFACES
$E(\xi)=\int_P e(\xi)(u)dP$
$(\sqrt{\det[g_{ij}]}du^1du^2)$
$g^{ij}(u)\partial\xi^\alpha(u)/\partial u^i\partial\xi^\beta(u)/\partial u^j h_{\alpha\beta}(\xi(u))$
$0=L(\xi^i)=\sum_{m=1}^{2}\partial/\partial u^m[\det(g^{ru})\sum_{l=1}^{2}g^{ml}_{ur}\partial\xi^i/\partial u^l]$
$\xi^\alpha(u)$
LEAST DISTORTION
PARAMETERIZATION-INVARIANT
LEVEL SET IMPLEMENTATION
$E[\vec{u}]\triangleq\int_{\mathcal{M}}e[\vec{u}]\,d_{\mathcal{M}}v$
$e[\vec{u}](x)\triangleq\frac{1}{2}g^{pq}(x)\,h_{ij}(\vec{u}(x))\,\frac{\partial u^i}{\partial x_p}\frac{\partial u^j}{\partial x_q}.$
E-L:
$\Delta_{\mathcal{M}}u^l+\Gamma^l_{ij}(\vec{u})\,g^{\alpha\beta}\frac{\partial u^i}{\partial x^\alpha}\frac{\partial u^j}{\partial x^\beta}=0$
Gradient Descent, Onto Level Set:
$\frac{\partial u^i}{\partial t}=\Delta u^i+\sum_{k=1}^{d}\mathbf{H}_\psi(\vec{u})\left[\frac{\partial\vec{u}}{\partial x_k},\frac{\partial\vec{u}}{\partial x_k}\right]\frac{\partial\psi}{\partial u^i}(\vec{u})$

Nonlinear Loss (*N*=176 normal subjects)



Sowell/Peterson/Thompson/Welcome et al., 2003, Nature Neuroscience



Sowell/Peterson/Thompson/Welcome et al., 2003

Drug Trial Example:

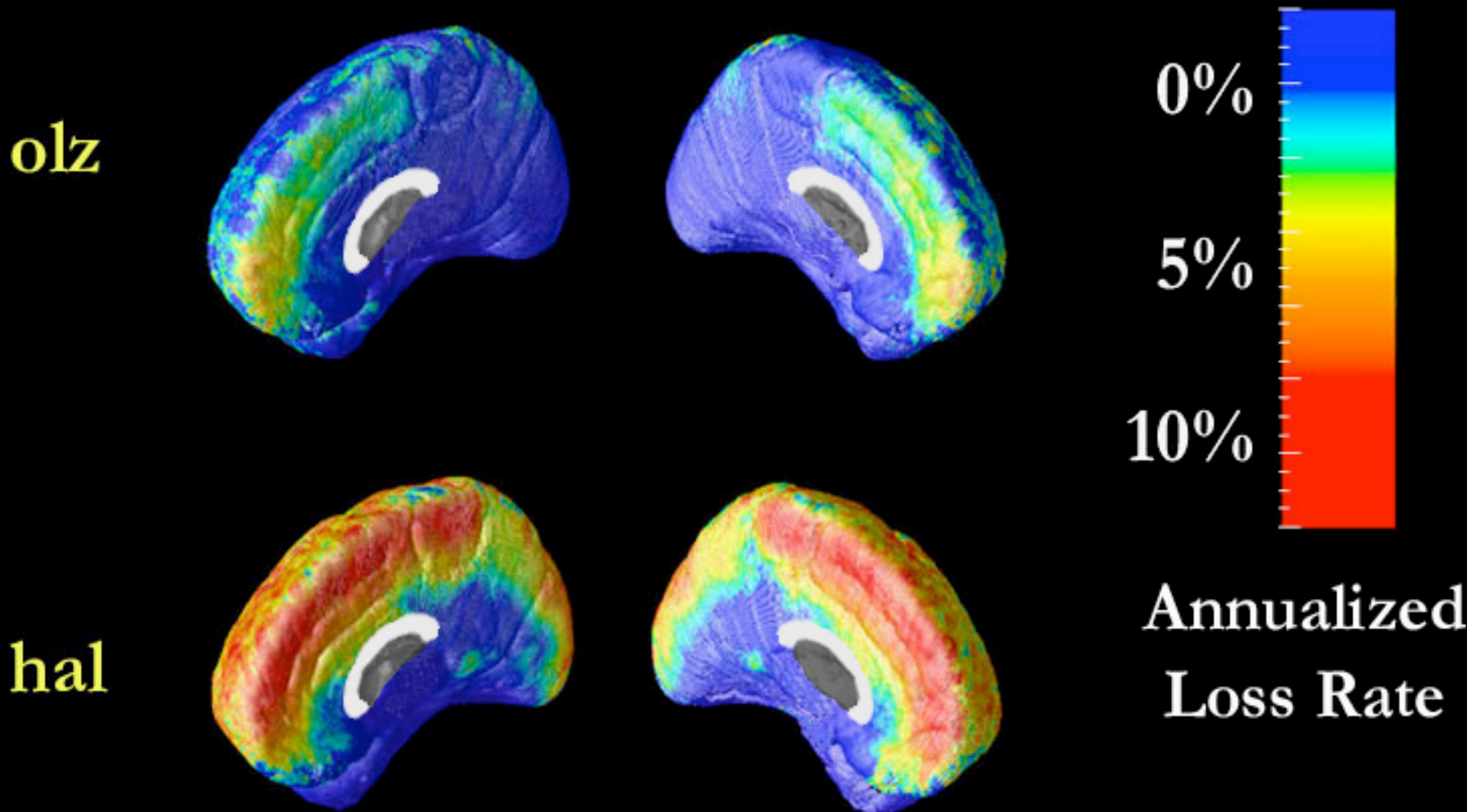
Olanzapine (atypical)
vs.
Haloperidol (typical)

Thompson/Lieberman

Lilly HGDH Study (2007)

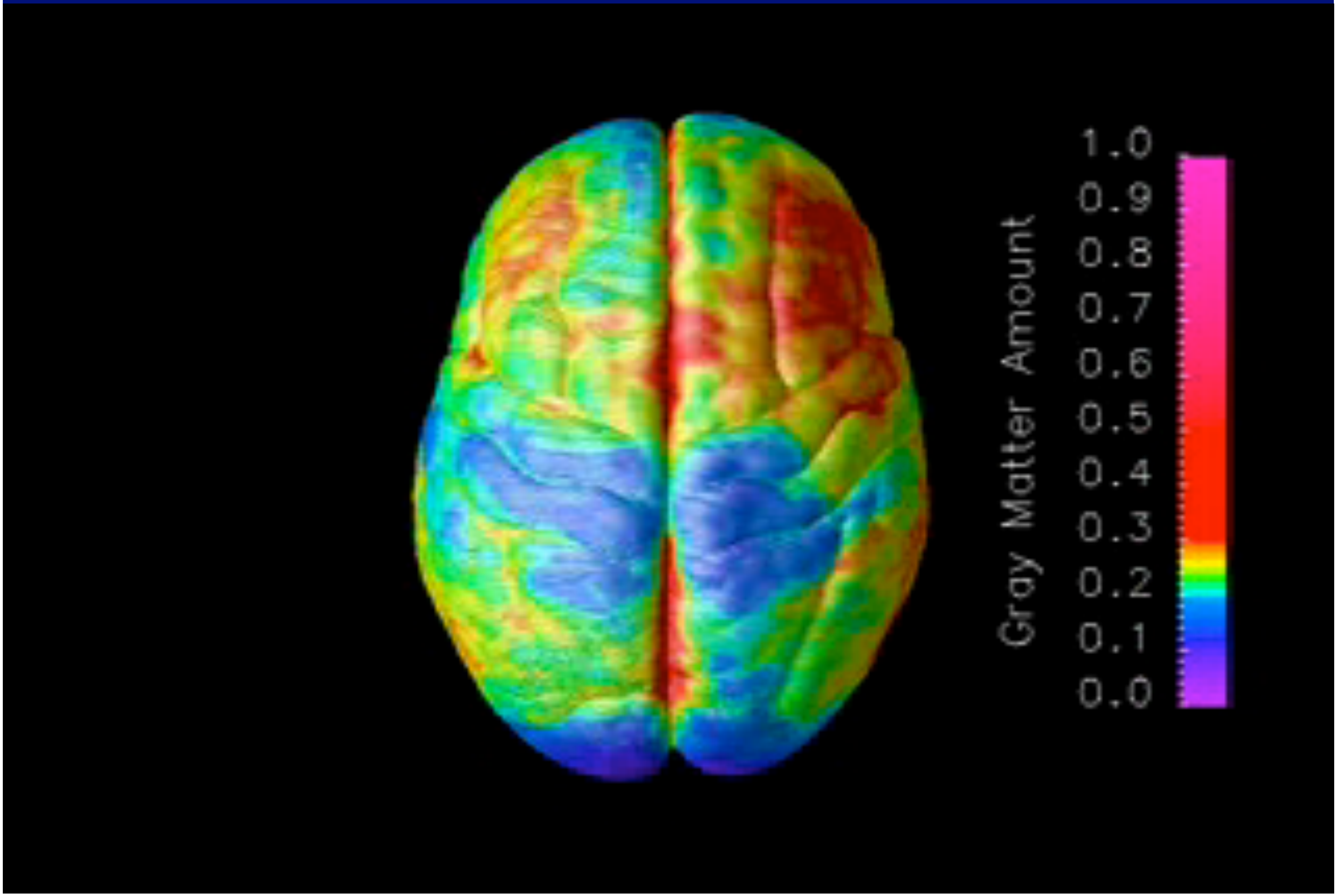
Do Antipsychotics Resist Gray Matter Loss in Schizophrenia?

Comparing Drugs: Olanzapine Slows the Rate of Gray Matter Loss



Thompson/Lilly-HGDH/Lieberman 2007

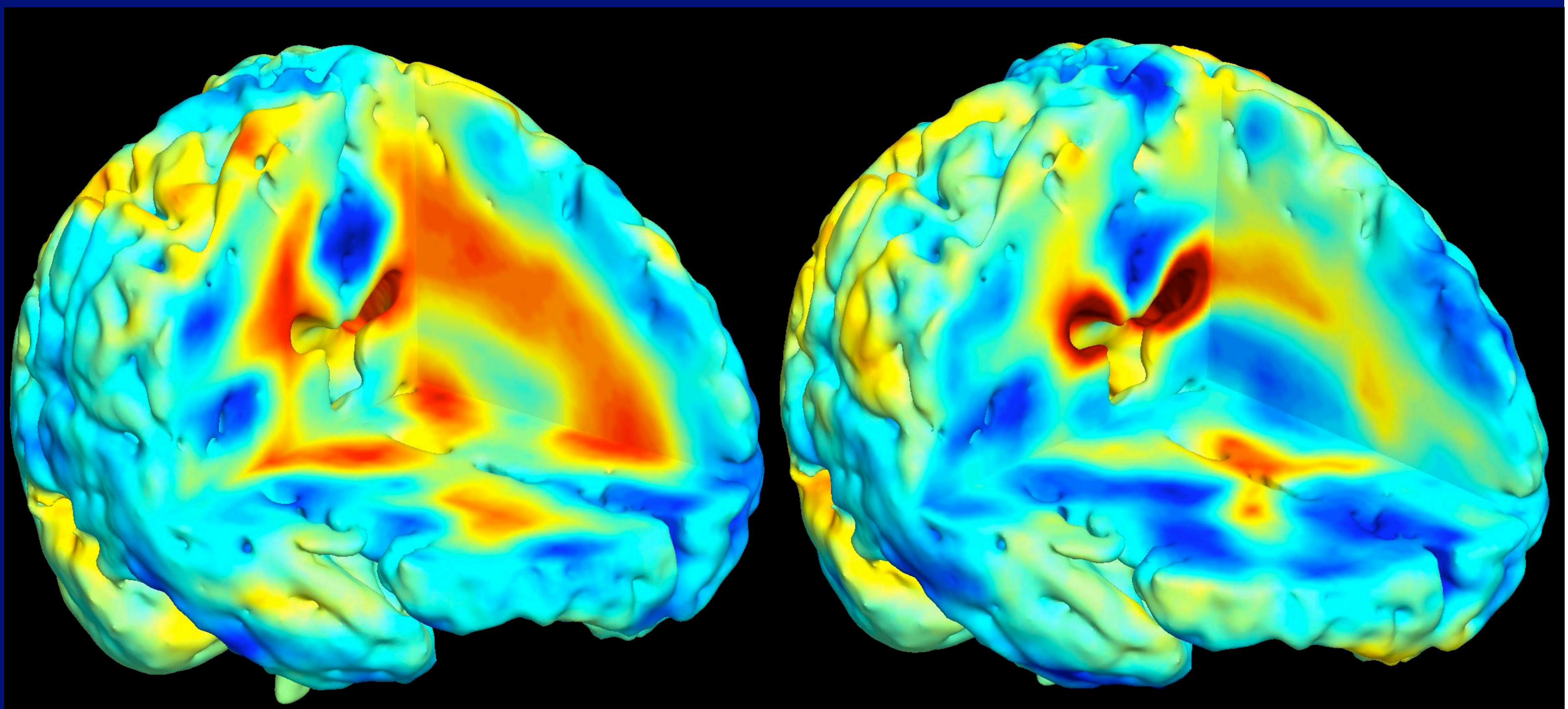
Time-Lapse Map of Brain Development (PNAS, 2004)



Mean Annualized Growth Rate Maps

NIMH HEALTHY CONTROLS
N=12 (6 male / 6 female), AGE: 13.5±0.7 yrs. at baseline

NIMH CHILDHOOD-ONSET SCHIZOPHRENIA (COS)
N=12 (6 male / 6 female), AGE: 13.9±0.8 yrs. at baseline

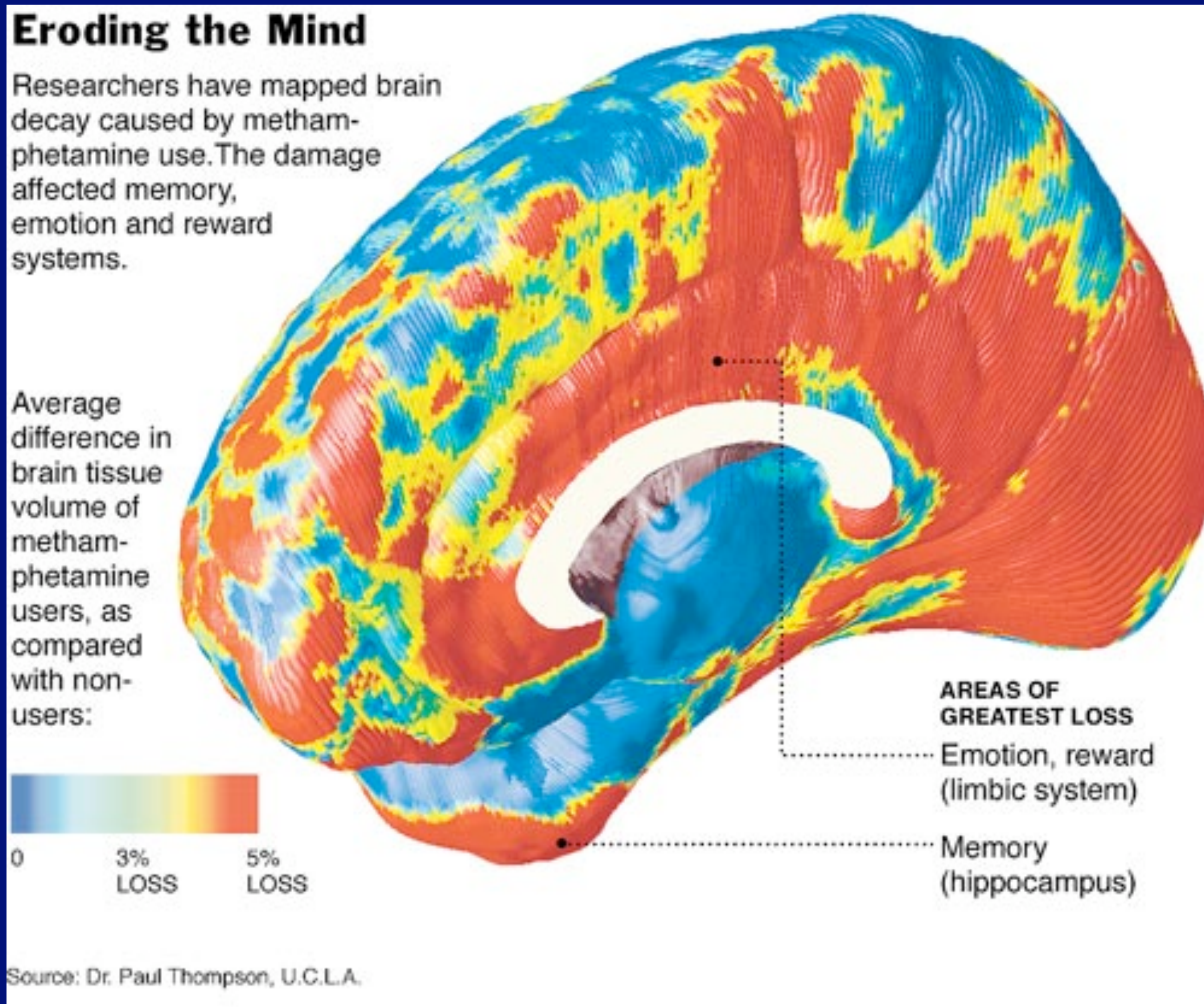


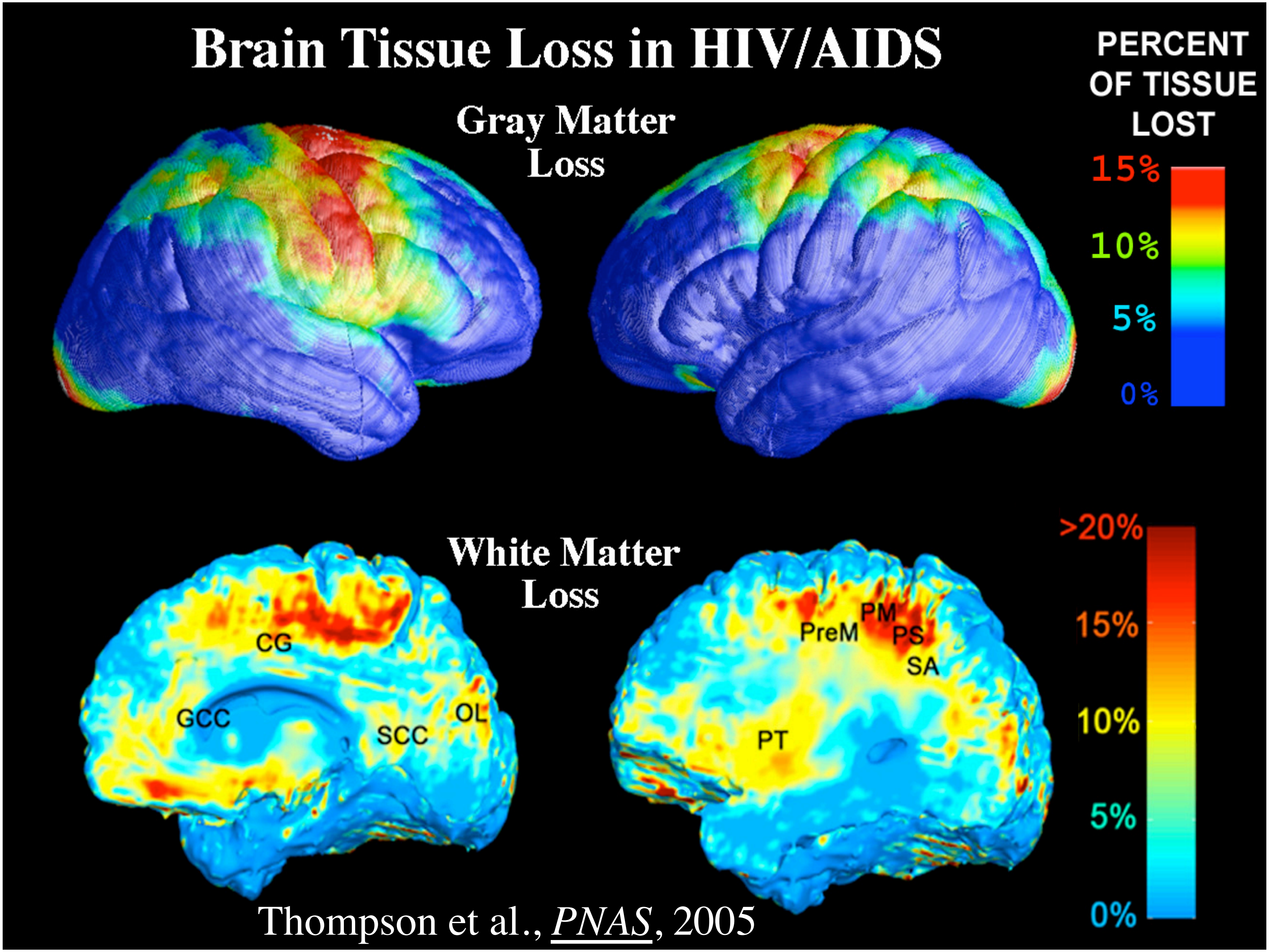
% Growth Rate shown in RED
% Loss Rate shown in BLUE

Methamphetamine Brain Damage (Thompson et al., *J Neurosci*, 2004)

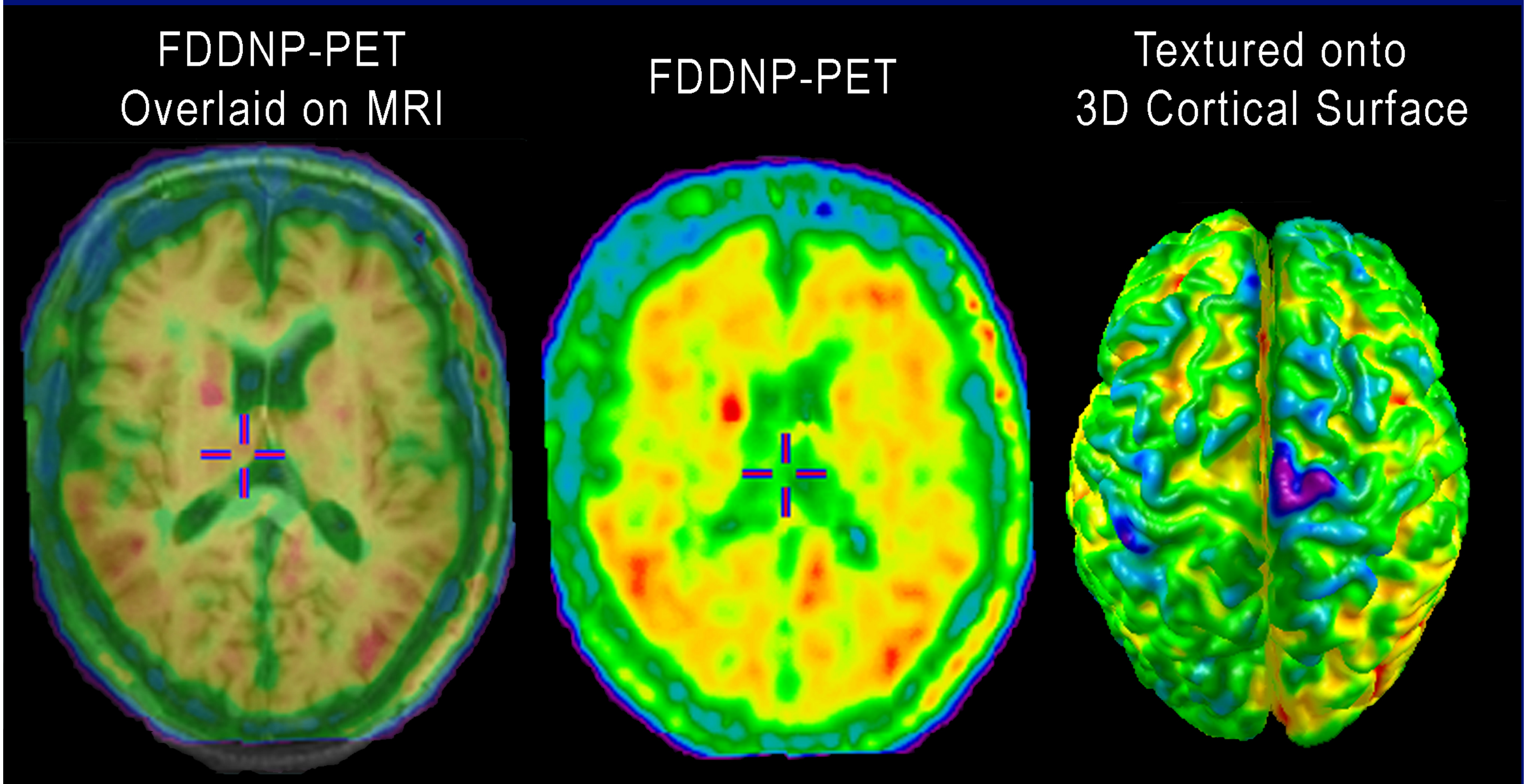
22 chronic meth abusers and 21 Controls scanned with MRI

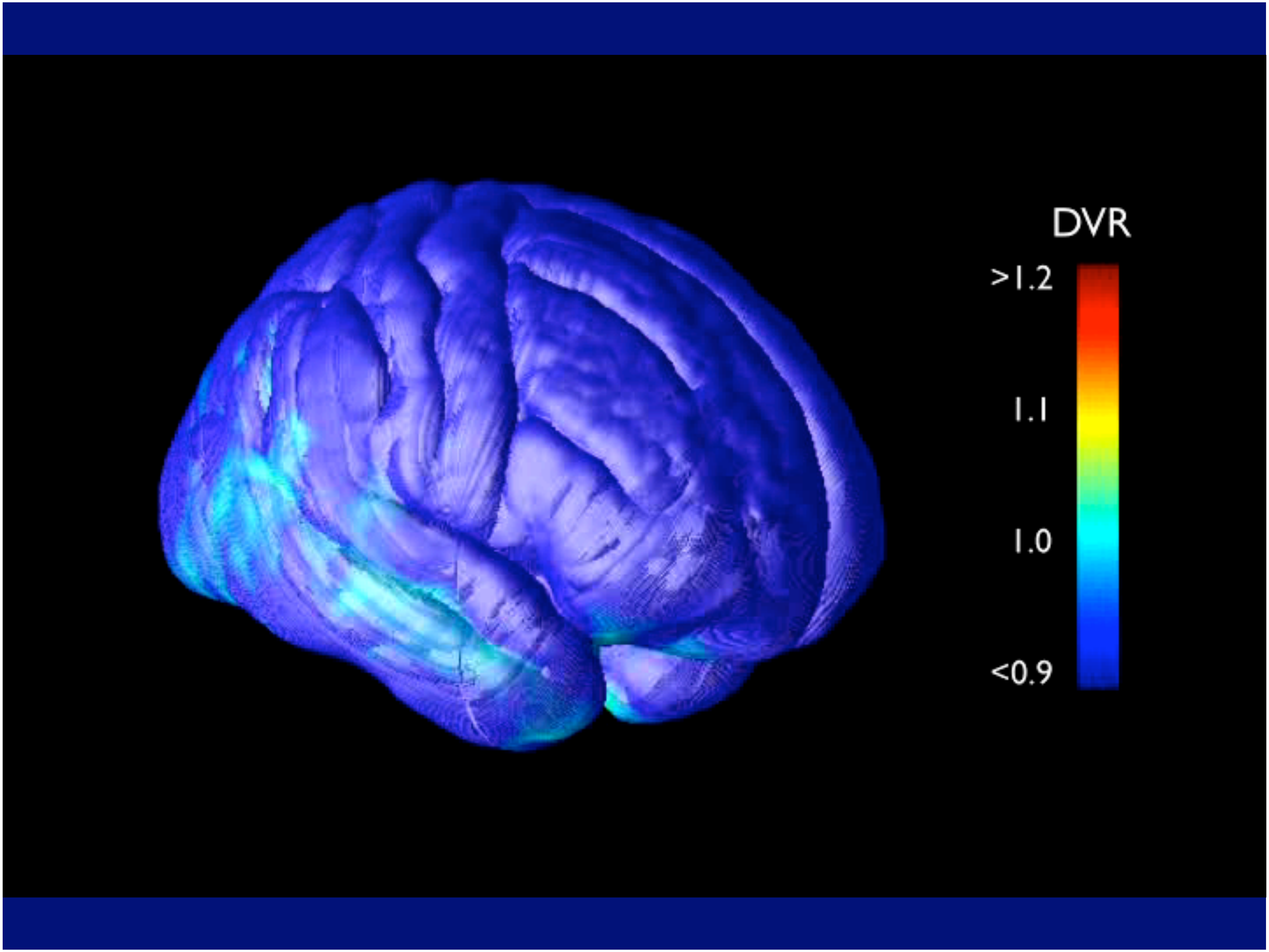
What is the profile of cortical thinning?

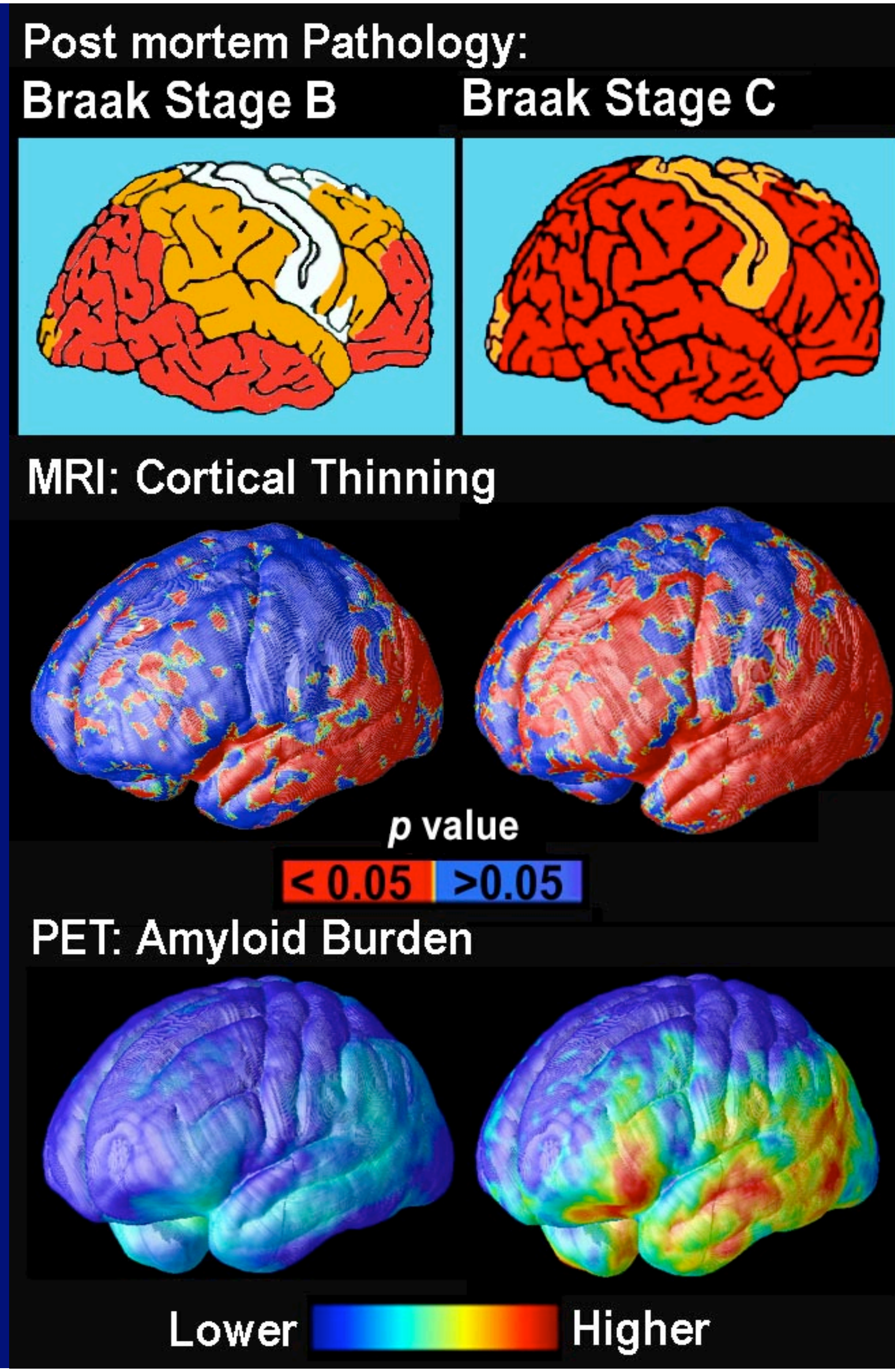


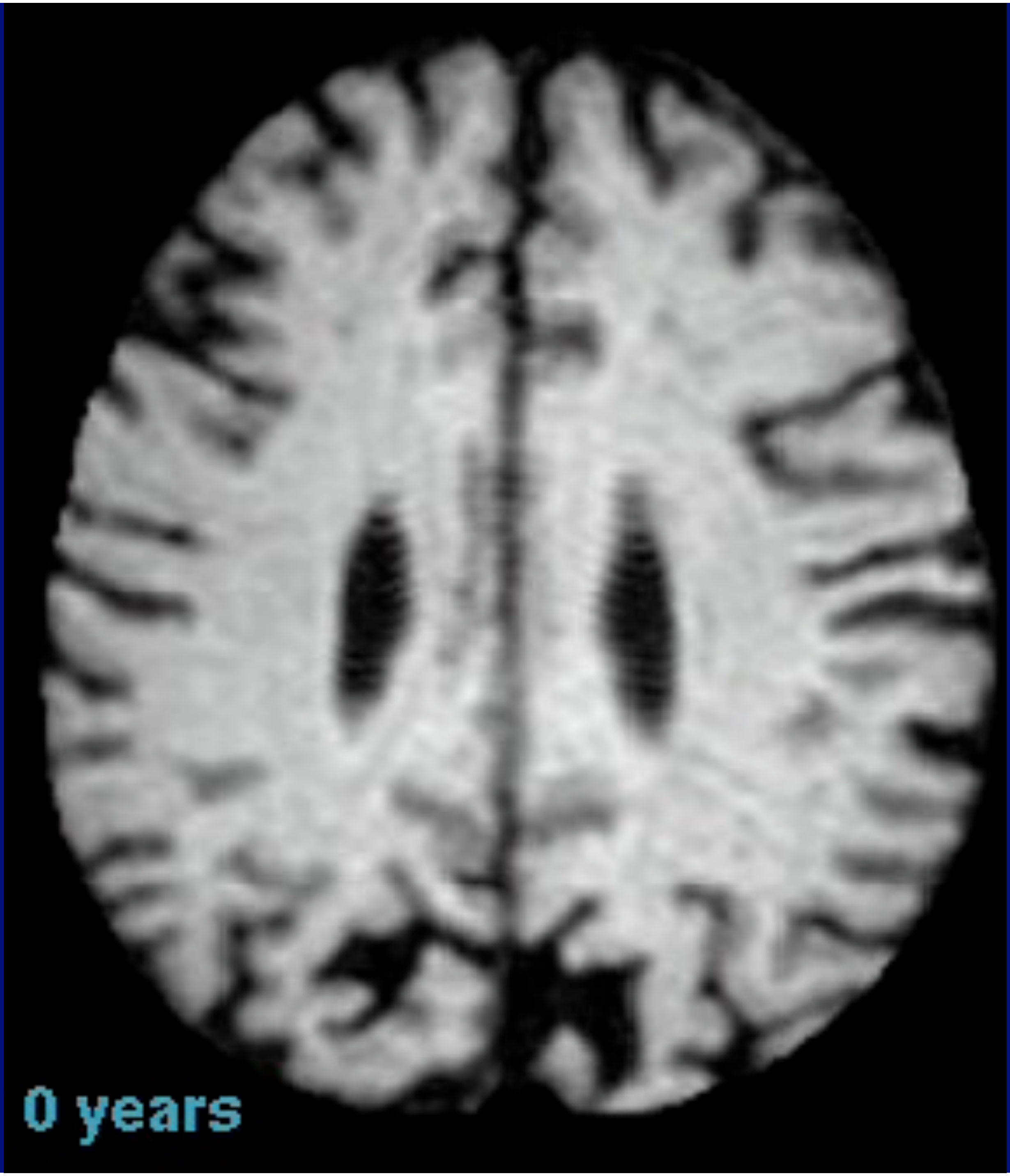

Brain Tissue Loss in HIV/AIDS
Gray Matter Loss
PERCENT OF TISSUE LOST
15%
10%
5%
0%
White Matter Loss
CG
GCC
SCC
OL
PreM
PM
PS
SA
PT
>20%
15%
10%
5%
0%
Thompson et al., PNAS, 2005

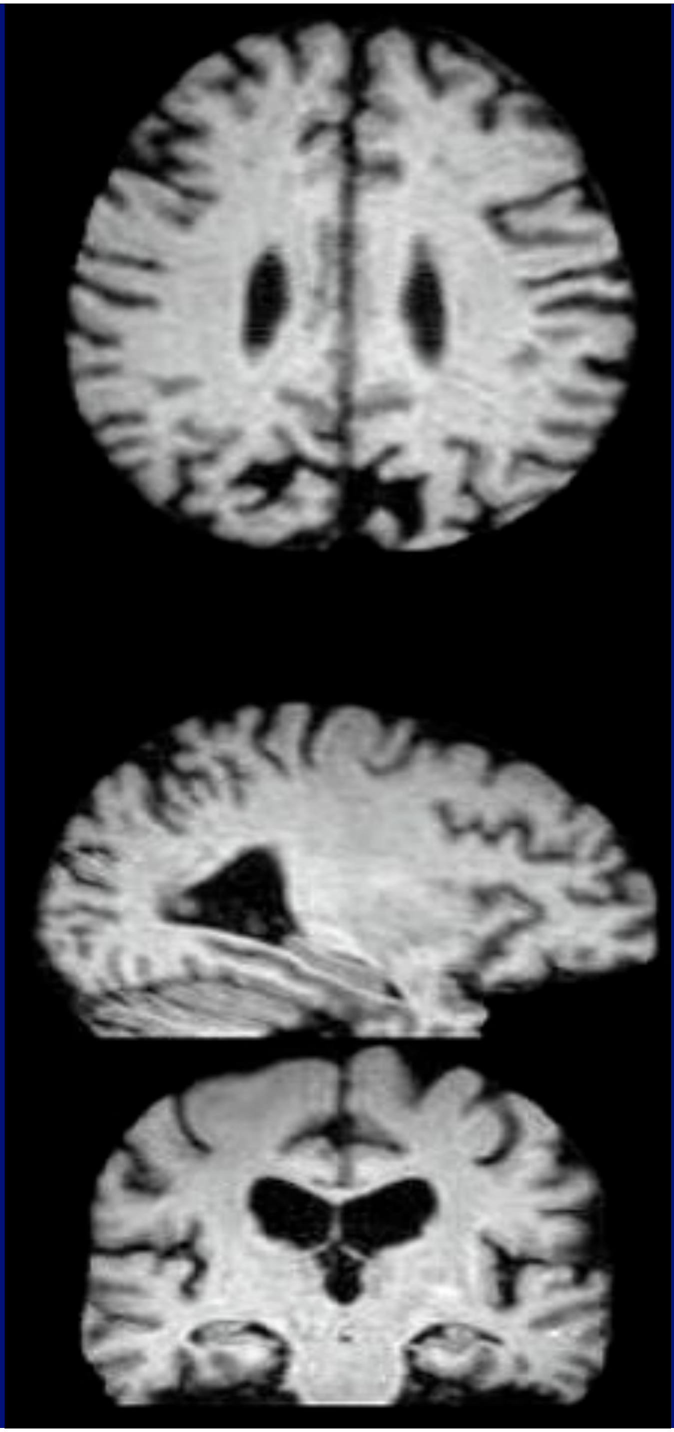
[redacted]


FDDNP-PET
Overlaid on MRI
FDDNP-PET
Textured onto
3D Cortical Surface


DVR
>1.2
1.1
1.0
<0.9


Post mortem Pathology:
Braak Stage B
Braak Stage C
MRI: Cortical Thinning
p value
< 0.05
>0.05
PET: Amyloid Burden
Lower
Higher


0 years



Fluidly Deform One 3D Image to Match Another, Maximizing the Mutual Information

Mutual Information (MI):

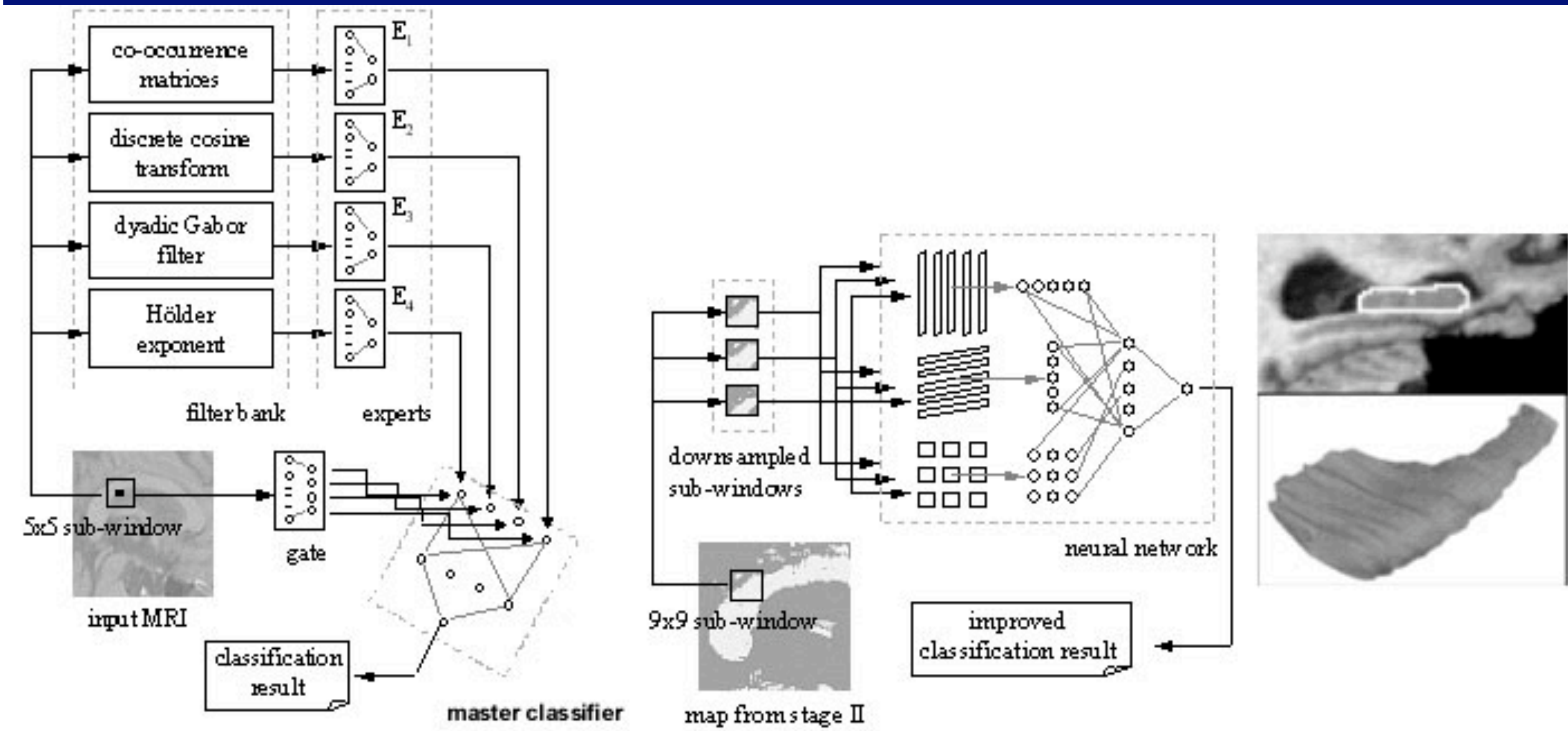
$$\mathbf{MI}(\mathbf{u}) = \int_{\mathrm{R}^2} p_{\mathbf{u}}(i_1, i_2) \log \frac{p_{\mathbf{u}}(i_1, i_2)}{p(i_1)\, p_{\mathbf{u}}(i_2)} di_1 di_2 ,$$

Use the gradient of the $\mathbf{MI}$ to drive the velocity field of the fluidly flowing image:

$$\boldsymbol{L}\mathbf{v} = \mu \nabla^2 \mathbf{v} + (\lambda + \mu) \vec{\nabla}(\vec{\nabla} \cdot \mathbf{v}) = \mathbf{F}(\mathbf{x}, \mathbf{u})$$

$$= \vec{\nabla}\ \mathbf{MI}(\mathbf{u})$$

Computer Vision - Automatically Measure Brain MRI



Terzopoulos, Yuille, Osher at UCLA – world leaders, we have existing collabs, shared PhD students