GEOMETRIC METHODS IN IMAGING APPLICATIONS: A SURVEY OF RECENT RESULTS

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Thanks to National Science Foundation, Office of Naval Research, and the Department of Defense.

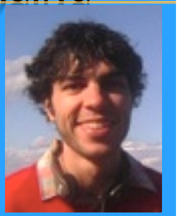
Collaborators: Stefano Soatto (UCLA CS), Alexander Tartakovsky (USC Math), Stan Osher (UCLA Math), Arjuna Felnner (China Lake) and many students and postdocs

FORMER AND CURRENT POSTDOCS-IMAGING


Selim Esedoglu
U Mich - tenure


Marc Droske
Industry


John Greer
(Courant and NGA)


George Mohler
Santa Clara Univ.
Tenure track


Xiaoqun Zhang
Shanghai Jiaotong U
Professor


Mario Micheli
(current)


Yves van Gennip
(current)


Todd Wittman
(current)

IMAGING STUDENTS SUPERVISED AT UCLA


Alan Gillette PhD '06
(business owner)


Julia Dobrosotskaya
PhD '09
(postdoc UMD)


Carola Schoenlieb
PhD (Cambridge) '09
(tenure track Cambridge)


Laura Smith
current


Matthew Keegan
Current - Chan


Michael Moeller
MS Muenster '09
Current PhD student


Yifei Lou PhD
'10 (postdoc
GA Tech)

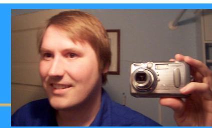

Alex Chen current

Wenhua Gao current

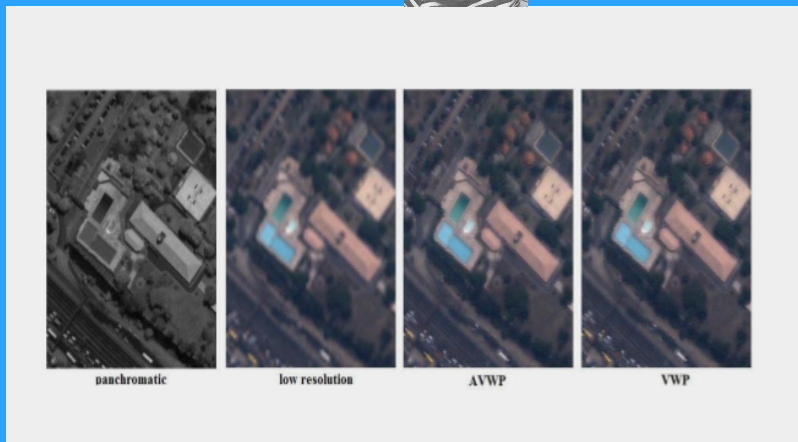
OUTLINE

- Pan Sharpening
- Hyperspectral Sharpening
- Density estimation and data fusion
- Diffuse interface methods for inpainting and deblurring
- Diffuse interfaces on graphs
- Convex Splitting for fast computation
- Curvature dependent geometric curve evolution

PAN SHARPENING



Moeller, Wittman, Bertozzi



RECENT PANSHARPENING TECHNIQUES

- IHS
- Brovey
- PCA
- Wavelet Fusion
- First variational approach: 'A Variational Model for P+XS Image Fusion', Ballester, Casselles, Igual, Verdera, 2006

PAN SHARPENING

Assumes panchromatic is a linear combination of spectral bands.

$$u_i = \uparrow M_i + (Panchromatic - Intensity)$$

$$Intensity = \frac{1}{4}\sum \uparrow M_i$$

FULL VWP VARIATIONAL PROBLEM

The total energy functional then is

$$\begin{aligned} J(X_n) = & \sum_{n=1}^{N} \gamma \int_\Omega |\nabla u_n|\, dx + \eta \int_\Omega \mathrm{div}(\theta) \cdot u_n \, dx \\ & + \sum_n c_0 (a^i_L[n] - \alpha^i_L[n])^2 \phi^2_{L,n} \\ & + \sum_n \sum_{j=1}^{L} \sum_{k=1}^{3} c_j (d_{\{k,j\}}[n] - \beta^i_{\{k,j\}}[n])^2 \psi^k_{j,n} \\ & + \mu \sum_{i=1, i<j}^{N} \int_\Omega (u_i \cdot \uparrow M_j - u_j \cdot \uparrow M_i)^2 \, dx \\ & + \nu \sum_{i=1}^{N} \int_{\Omega - \Gamma} (u_i - \uparrow M_i)^2 \, dx \end{aligned}$$

ALTERNATE VWP – AVOIDS SWITCHING FROM WAVELET TO PHYSICAL SPACE

Denoting the wavelet fused image for the i^{th} band with W_i and the new matching image with Z_i we define

$$Z_i = exp(-\frac{d}{|\nabla P|^2}) \cdot W_i + (1 - exp(-\frac{d}{|\nabla P|^2})) \cdot \uparrow M_i \qquad (20)$$

and combine the wavelet and the low resolution matching term to a fidelity term with Z_i. The alternate energy becomes

$$\begin{aligned} J(X_n) &= \sum_{n=1}^{N} \gamma \int_{\Omega} |\nabla u_n| \, dx + \eta \int_{\Omega} div(\theta) \cdot u_n \, dx \\ &+ \mu \sum_{i=1, i<j}^{N} \int_{\Omega} (u_i \cdot \uparrow M_j - u_j \cdot \uparrow M_i)^2 \, dx \\ &+ \nu \sum_{i=1}^{N} \int_{\Omega} (u_i - Z_i)^2 \, dx \qquad (21) \end{aligned}$$

EXTENSION TO HYPERSPECTRAL DATA

Dataset AVIRIS San Diego Harbor Hyperspectral image

MASTER IMAGE OBTAINED FROM GOOGLE

master image | low resolution | VWP sharpened

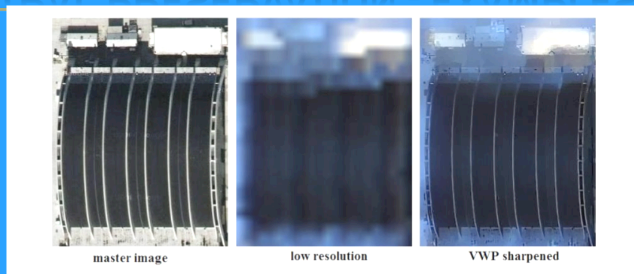
Master image from Google Maps©.

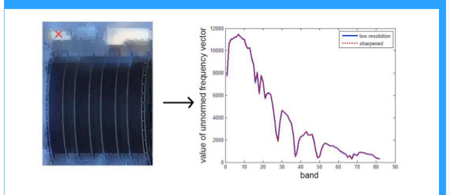
SPATIAL DETAIL INHERITED FROM MASTER IMAGE – SPECTRAL DETAIL FROM AVIRIS DATA

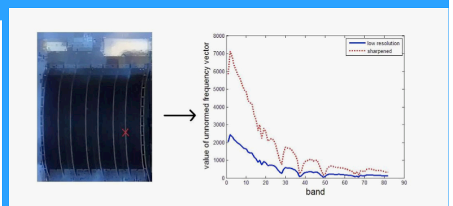
master image | low resolution | VWP sharpened

Master image from Google Maps©.

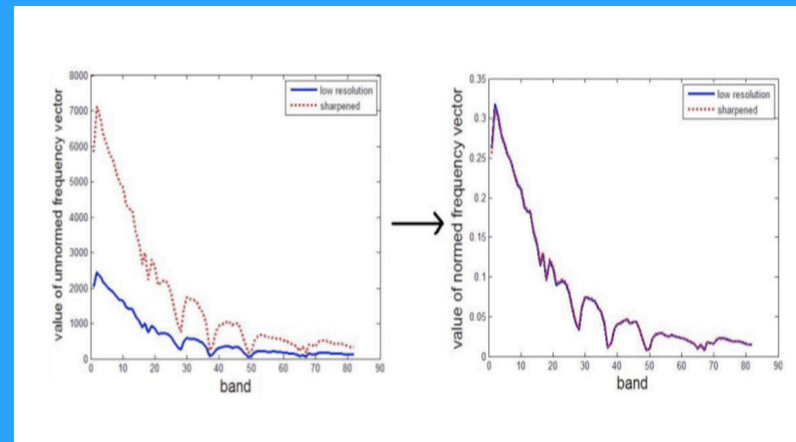
SPECTRAL PRESERVATION - EXAMPLES





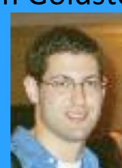


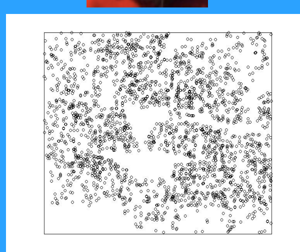
SPECTRAL ANGLE IS PRESERVED



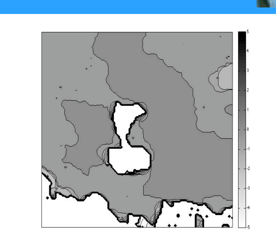
Maximum Penalized Likelihood Estimation

George Mohler, Andrea Bertozzi, Tom Goldstein, Stan Osher

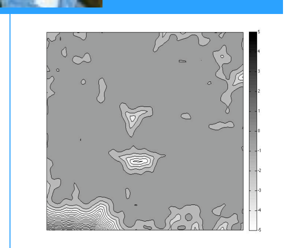




Point process data



Density using TV



Density using kernels

Data are residential burglaries in the San Fernando Valley (courtesy of LAPD)
Repeated solves of TV minimization – requires fast compressive sensing algorithms

Maximum Penalized Likelihood Estimation basic problem

- Estimate probability density u(x) from point data x_1, x_2, x_3, etc.
- General approach for regularizer R(u).

$$u(x) = \arg\max_{u(x)} \{ \sum_{i=1}^{N} \log(u(x_i)) - \alpha R(u) \}$$

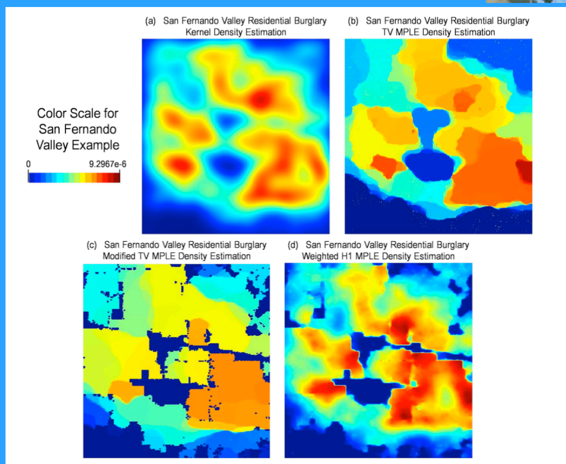
- For discontinuous densities, choose R = TV

$$u(x) = \arg\max_{u(x)} \{ \sum_{i=1}^{N} \log(u(x_i)) - \alpha \int |\nabla u(x)| \}$$

DENSITY ESTIMATION USING DATA FUSION

LAURA SMITH, MATTHEW KEEGAN UCLA

L. M. Smith, M. S. Keegan, . Wittman, G. O. Mohler and A. L. Bertozzi, *EURASIP J. on Advances in Signal Processing*, 2010.



DIFFUSE INTERFACE METHODS

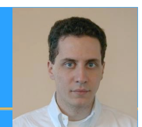
$$\int |\nabla u| dx \sim \frac{\epsilon}{2} \int |\nabla u|^2 + \frac{1}{\epsilon} \int W(u) dx$$

Total variation Ginzburg-Landau functional

$$\Delta u = -4\pi^2 \sum_k k^2 < u, e^{-2\pi i k \cdot x} > e^{-2\pi i k \cdot x}$$

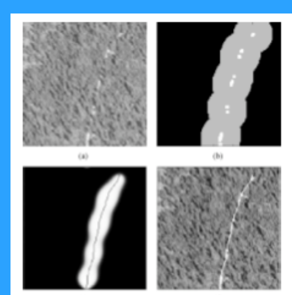
$$\Delta_w u := -\sum_j 2^{2j} < u, \psi_{j,k} > \psi_{j,k}$$

CAHN-HILLIARD INPAINTING



US Patent No. 7,840,086

Bertozzi, Esedoglu, Gillette, *IEEE Trans. Image Proc. 2007, SIAM MMS 2007*
Transitioned to NGA for road inpainting.
Transitioned to InQtel for document exploitation.
Continue edges in the same direction – higher order method for local inpainting.
Fast method using convexity splitting and FFT



$$u_t = -\nabla^2 \left(\varepsilon \nabla^2 u - \frac{1}{\varepsilon} W'(u) \right) + \lambda(\vec{x})(f - u)$$

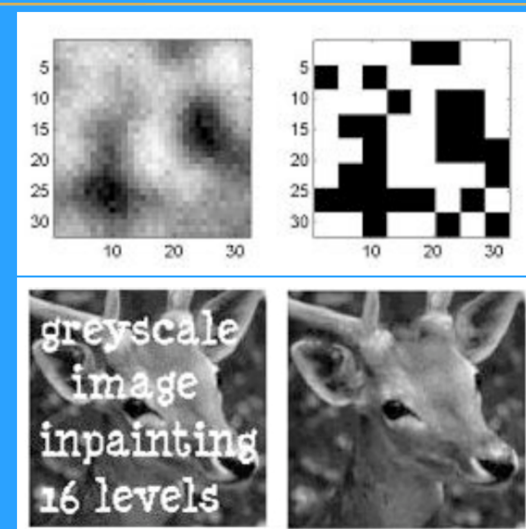
where

$$\lambda(\vec{x}) = \begin{cases} 0 & \text{if } \vec{x} \in D \\ \lambda_0 & \text{if } \vec{x} \in \Omega \setminus D. \end{cases}$$

H^{-1} gradient flow for diffuse TV
L^2 fidelity with known data

WAVELET ALLEN-CAHN IMAGE PROCESSING

- Dobrosotskaya, Bertozzi, *IEEE Trans. Image Proc. 2008, Manuscript accepted Interfaces and Free Boundaries.*
- *Transitioned to NGA for road inpainting. Transitioned to InQtel for document exploitation.*
- Nonlocal wavelet basis replaces Fourier basis in classical diffuse interface method.
- Analysis theory in Besov spaces.
- Gamma convergence to anisotropic TV.



GAMMA CONVERGE OF WAVELET GINZBURG-LANDAU ENERGY

Dobrosotskaya and Bertozzi, IFB 2011

$$GL_\epsilon(f) = \frac{\epsilon}{2}\int |\nabla f(x)|^2 dx + \frac{1}{4\epsilon}\int W(f(x))dx, \quad W(f) = (f^2-1)^2$$

$$WGL_\epsilon(f) = \frac{\epsilon}{2}|f|_B^2 + \frac{1}{4\epsilon}\int W(f(x))dx, f \in H^1,$$

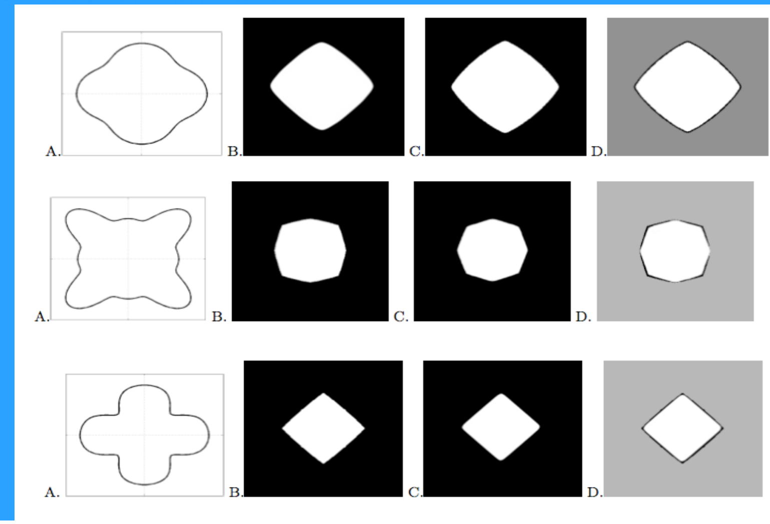
Theorem:

$$WGL_\epsilon(u_\epsilon) \xrightarrow{\Gamma} G_\infty(u), \; G_\infty(u) = \frac{\sqrt{2}}{3}C(u)|u|_{TV},$$

$$G_\infty(\chi_E) = \int_{\partial E} \rho(\vec{n}(x), \psi)dl(x),$$

GAMMA CONVERGE OF WAVELET GINZBURG-LANDAU ENERGY

Dobrosotskaya and Bertozzi, IFB 2011

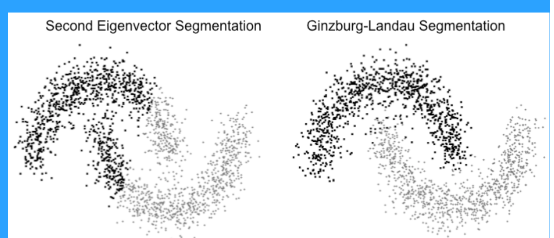


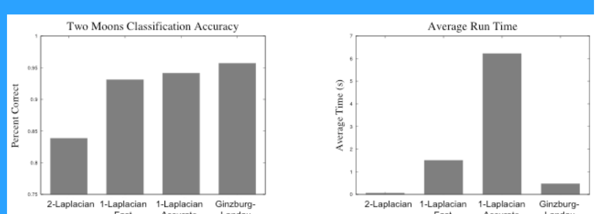
DIFFUSE INTERFACES ON GRAPHS

Joint work with Arjuna Flenner

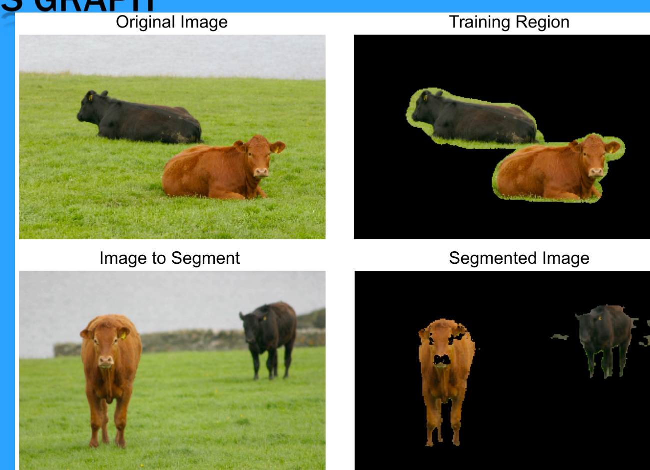
Replaces Laplace operator with a weighted graph Laplacian in the Ginzburg Landau Functional

Allows for segmentation using L1-like metrics due to connection with GL





DIFFUSE INTERFACES ON GRAPHS – NONLOCAL MEANS GRAPH



High dimensional fully connected graph – use Nystrom extension methods for fast computation methods.

CONVEX SPLITTING SCHEMES

Schoenlieb and Bertozzi, *Comm. Math. Sci. 2011*

Basic idea:

$$E(u) = E_c(u) - E_e(u)$$

$$U_{k+1} - U_k = -\Delta t(\nabla E_c(U_{k+1}) - \nabla E_e(U_k))$$

Art is to choose Ec to give an implicit problem that is easy to solve
- e.g. Ec is H1 semi norm – can be solved using FFT
- in wavelet case Ec is wavelet Laplace operator

Contraints on Ec and Ee so that splitting is unconditionally stable

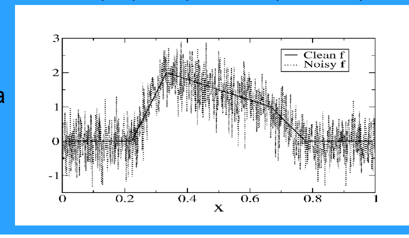
Proof of convergence of splitting schemes for various higher order inpainting methods.

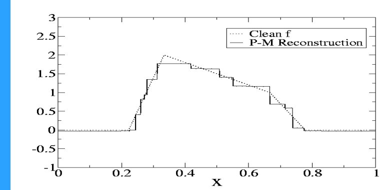
DENOISING PIECEWISE LINEAR SIGNALS

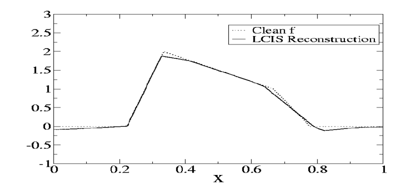
$$u_t = -\nabla \cdot (g(\Delta u)\nabla \Delta u) + \lambda(f - u)$$

ALB. And Greer *CPAM* 2004

Noisy data

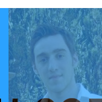






Perona-Malik

LCIS-based method



Marc Droske and Wenhua Gao

SEGMENTATION WITH CORNERS

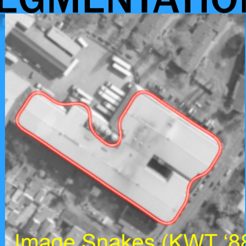
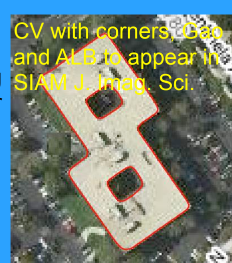


Image Snakes (KWT '88)

Chan-Vese 2001

Droske & Bertozzi – geometric corner snakes 2010-SIAM Im. Sci.

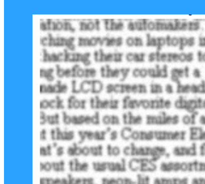


CV with corners, Gao and ALB to appear in SIAM J. Imag. Sci.

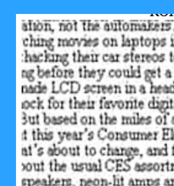
Idea – segmentation requires a regularization
It is analogous to denoising.
CV, Snakes reduce length of curve.
Removes corners as well as noise.
Instead regularize with the "curve" analogy of TV – nonlinear penalization of curvature-based functional.
Extend to curve evolution using either Lagrangian framework or Level sets.
Extends LCIS PDE to an intrinsic geometric based motion – a nonlinear form of surface diffusion.

Lou, Bertozzi, Soatto, JMIV 2010

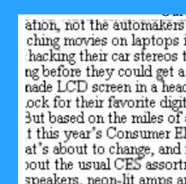
DIRECT SPARSE DEBLURING



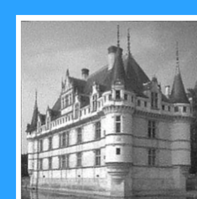
Blurry data



ROF deblurring



Our method



Uses training data

Dictionary based

Inverse problem not solved

Fit data to blurred dictionary then directly unblur

Solves problem of amplifying noise with solution of inverse problem

PREPRINTS

George O. Mohler, Andrea L. Bertozzi, Thomas A. Goldstein, and Stanley J. Osher Fast TV regularization for 2D Maximum penalized likelihood estimation, to appear in J. Comp. Geo. Statistics, 2010 .

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W. Gao and A. L. Bertozzi, Level set based multispectral segmentation with corners, to appear in SIAM J. Imag. Sci., 2011.

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PUBLICATIONS

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Julia A. Dobrosotskaya and Andrea L. Bertozzi, Interfaces and Free Boundaries, 12(2), 2010, pp. 497-525.

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Andrea Bertozzi, Selim Esedoglu, and Alan Gillette, Inpainting of Binary Images Using the Cahn-Hilliard Equation, IEEE Trans. Image Proc. 16(1) pp. 285-291, 2007.